

Analysis of Quarterly Revisions of Fixed Quantity Consumer Price Index Products

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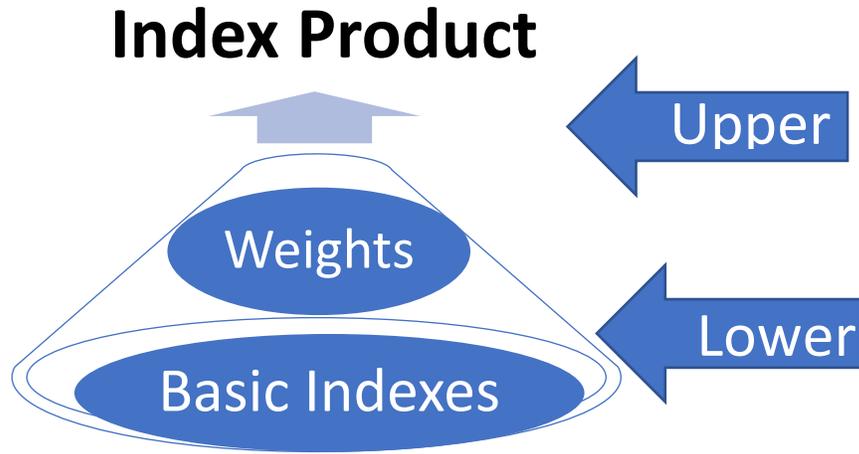
Abstract

The U.S. Bureau of Labor Statistics Consumer Price Index (CPI) program revises fixed quantity weights for products such as the CPI-U, CPI-W, and preliminary C-CPI-U on a biennial basis. Previous work explored annual revisions to improve the timeliness of fixed quantity weights by using household expenditures from a more recent reference period from the Consumer Expenditure Survey. The following evaluates revisions at a quarterly frequency to reduce the lag when the household expenditure weights are first used in the index, to reduce upper-level substitution bias as reflected by fixed quantity weight index products. Quarterly expenditure weights represent a subset of annual weights. Therefore, smoothing mechanisms are used to improve elementary item-area cell coverage and reduce variability of quarterly surveyed expenditure estimates. The risk of chain drift is evaluated due to the seasonality reflected in the quarterly expenditure weights. Lastly, analysis of quarterly revisions as four quarter rolling sums are evaluated as an alternative to the existing calendar year expenditure weight revisions.

Key Words: CPI-U, index weight revisions, Consumer Expenditure Survey

1. Introduction

The Consumer Price Index for the urban population (CPI) and the chained version (CCPI) are measures of consumer price inflation (Bureau of Labor Statistics, 2020, Handbook of Methods, Consumer Price Index). Index estimation is divided into a lower-then upper-level processing stages as depicted in Graph 1. Lower-level processing calculates basic item-area indexes that are derived from prices and calculates corresponding market basket weights derived from the Consumer Expenditure (CE) household survey. The same lower-level basic indexes are used for CPI products. Upper-level aggregate index formulas weight these lower-level differently to create an Index Product.



Graph 1. Index estimation as lower- and upper-level stages

CPI products differ by the index formula and corresponding weights. The CPI Low formula, also referred to as a modified Laspeyres formula, is an arithmetic average where biennial weights are updated January of even years. These biennial weights lag the index by an average of 3 years. Section 2 - overview of weight revision timelines provide more detail. Reducing reference period weight lag, the time between when the household data is collected until index use, improves the weight representivity (Balk and Diewert 2003, Greenlees and Williams 2009, Huang et. al. 2015 and 2015, Klick 2021, United Nations Economic Commission for Europe 2021, Interagency Technical Working Group on Consumer Inflation measures 2021, National Academies of Sciences, Engineering, and Medicine, 2022). The annualized growth rate, or average change across years, from December 1999 to September 2021 (262 months) is 2.3% (Federal Reserve Bank of Dallas).

This formula is summarized below as cost weights, which is the sum of the product of indexes and weights. The month-to-month price change is the ratio of cost weights from the current and previous month. The $\hat{P}_{\alpha k} \hat{Q}_{\beta k}$ is the basic level aggregation weight. The α represents the index average for the reference period for the corresponding β biennial period weight. The \hat{Q} expenditure weights are smoothed via a composite estimation and raking process comparable to production. One example from the 7,776 basic index combinations, k , is milk in the Washington–Arlington–Alexandria core based statistical area (CBSA). An example of an aggregate index, j , is the All items U.S. city average index.

CPI cost weight Low formula:

$$CPI_{t,0}^{LO} = \sum_{k \in j} (IX_{tk} \times \hat{P}_{\alpha k} \hat{Q}_{\beta k})$$

$CPI_{t,0}^{LO}$ = CPI Lowe formula.

$k \in j$ = k basic indexes (243 Items X 32 Areas = 7776 combinations) are elements of j aggregate index such as the All items U.S. city average.

IX_{tk} = Basic (item and area) level index for period t and previous period index is $t-1$.

$\hat{P}_{\alpha k} \hat{Q}_{\beta k}$ = Fixed quantity weights as basic level aggregation weight.

The final CCPI Tornqvist formula is a geometric average, where the weights are a 2-month moving average for the corresponding index month. This final version is designed to approximate a cost-of-living-index (COLI) measurement objective more closely than the CPI (Cage et. al. 2003). A COLI concept answers, "...What expenditure level is needed to achieve a standard of living attained in a base period at current market prices" (Bureau of Labor Statistics, 2020, Handbook of Methods, Consumer Price Index). Monthly weights are available for index estimation approximately one year after publication of the CPI. The final CCPI is the goal, but the CPI cannot process this formula as a real time index due the lag of monthly weights. A preliminary version fills in gap with a Constant Elasticity of Substitution formula that is revised quarterly and chained the terminal month of the final CCPI (Klick 2018). The CCPI referenced in this paper is for final and will be notated as CCPI. The annualized growth for the CCPI from December 1999 to latest published month September 2021 is 2.0%, 0.3 percentage points less than the CPI.

$$CCPI_{t,0}^T = \prod_{k \in J} \left(\frac{IX_{tk}}{IX_{t-1k}} \right)^{\left(\frac{(\hat{P}\hat{Q})_{tk} + (\hat{P}\hat{Q})_{t-1k}}{(\hat{P}\hat{Q})_{tj} + (\hat{P}\hat{Q})_{t-1j}} \right) / 2}$$

$CCPI_{t,0}^T$

= Chained CPI Tornqvist formula. Chains together 1-month price changes

$(\hat{P}\hat{Q})$ = Estimated monthly expenditure weights for periods t and t-1 (2 month moving average) of basic index structure k divided by aggregate j

Upper-level substitution bias represents the difference between CPI Lowe formula which constrains consumer substitution due to relative price change, and the final CCPI Tornqvist formula, which reflects consumer substitution due to relative price change – where weights and indexes occur for the same month. There is a motivation to reduce this bias due to the fixed quantity lagged weights by improving the representivity of weights.

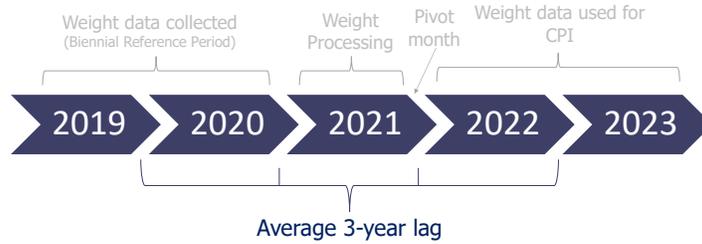
This paper builds upon work from last year which evaluated annual weight revisions. The following section reviews CPI fixed quantity weight revision timelines. Next, Section 3 is an analysis of the quality of quarter weights and the resulting All-items index. Section 4 presents additional quarter revision considerations. And then Section 5 presents takeaways and areas for future research.

2. Overview of weight revision timelines

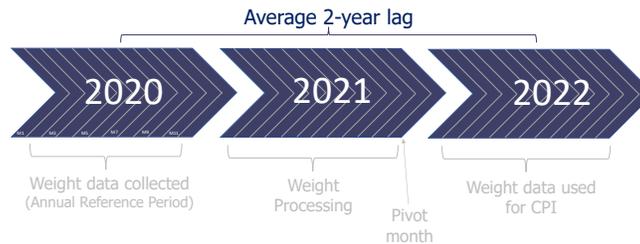
The current published CPI for 2022 is updated biennially, referenced as CPI-b going forward. This means weights from the 2019-2020 reference period are processed in 2021 and pivoted for index use beginning January 2022 resulting in an average 3-year lag notated as y-3. For an annually revised CPI referenced as CPI-a, 2022 indexes use weights from a 2020 reference period that are processed in 2021 resulting in an average 2-year lag notated as y-2. Beginning January 2023, the CPI plans to implement annual weight revisions to reduce the lag by 1 year (Bureau of Labor Statistics 2022 and 2022). For a quarterly revised CPI-q, 2022 1st quarter monthly indexes use quarter weights from

the 2021 1st quarter reference period resulting in a 1-year lag notated by y-1. The CPI-b, CPI-a, and CPI-q timelines are summarized in Graph 2.

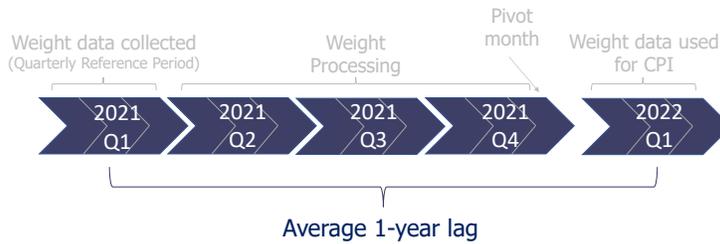
$$\text{CPI-b} = \hat{P}_{\alpha(y-3)} \hat{Q}_{\beta(y-3)}$$



$$\text{CPI-a} = \hat{P}_{\alpha(y-2)} \hat{Q}_{\beta(y-2)}$$



$$\text{CPI-q} = \hat{P}_{\alpha(y-1)} \hat{Q}_{\beta(y-1)}$$



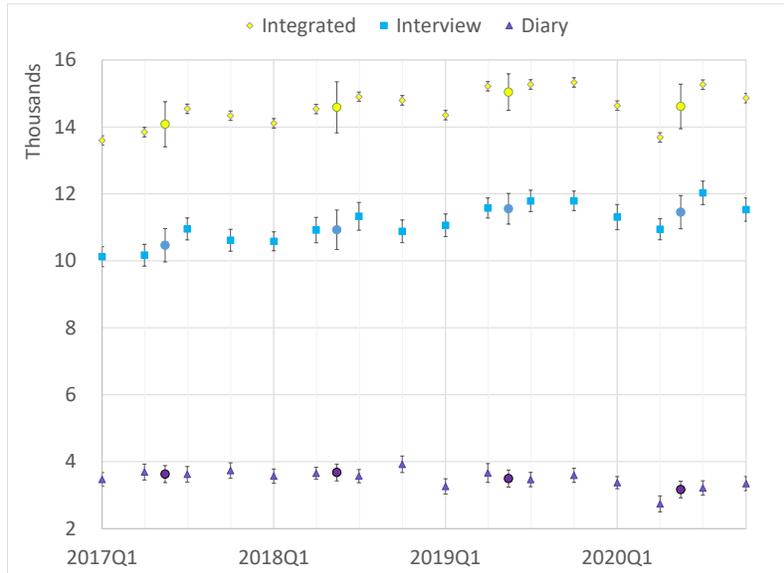
Graph 2. Weight revision timelines

3. Results

A. Data quality

The quality of urban household quarterly expenditure weights is subject to ¼ of annualized reported expenditures, and subject to quarterly seasonality. To evaluate the reduction in sample and corresponding seasonal expenditures, Graph 3 displays annual and quarterly average household expenditures from the CE Interview in blue and Diary in purple, as well as an integrated survey sources processed by the CPI in yellow as processed by the CPI. Annual estimates represented by the large circle and corresponding color in the middle of the year are divided by 4 to be comparable to quarter levels. Quarterly estimates include an interview month in quarter scope for the denominator population to adjust for the 6 collection months in scope, different than CE’s traditional adjustment for 15 collection months in scope for annual estimates (Bureau of Labor Statistics, Consumer Expenditure Survey, 2021). A comparison of the annual and quarter month in scope adjustments are summarized in Appendix A. The 95% confidence intervals use Balanced Repeated Replication comparable to CE methodology. The Diary quarterly average household expenditures display a modest 4th quarter peak, and the Interview and integrated sources display general 1st quarter trough. The trough in 2020 quarter 2 was due to Covid19 pandemic economic lockdown and recession (National Bureau of Economic Research, 2021). The quarterly household expenditure weights

appear relatively stable when compared to the time adjusted annualized household expenditures indicating sufficient quality.



Graph 3. Annual and quarterly average household expenditures for Diary and Interview surveys, and integrated sources

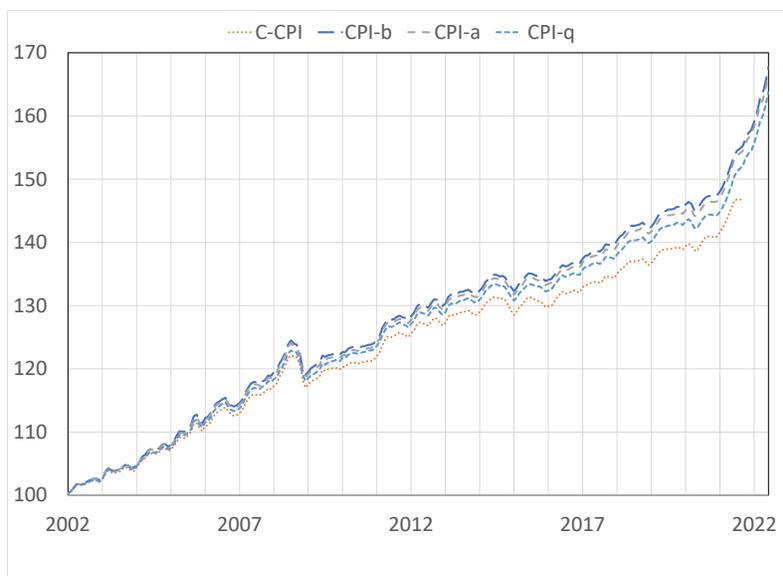
Effective 2018 forward, the market basket structure of weights consists of 243 items and 32 geographic areas resulting in a combined 7,776 item-area cells processed as weight upper-level index estimation. The medical care retained earnings items are allocated from 4 to 36 items. Analysis below occurs prior to this allocation to better represent collected expenditures of the 6,752 item-area combinations. Elementary cell coverage of weights is summarized in Graph 4 as cells where the collected expenditure weight is less than 1. The overall percent missing in blue is about 10%. Items can be further divided into priced and not priced, where prices are not collected due to cost or difficulty and therefore imputed at a higher item classification level such as unsampled owner’s equivalent of secondary residences, unsampled furniture, and unsampled apparel. The number of reported household expenditures for these not priced items is lower than for priced items and can be a misleading indicator of weight cell coverage as indicated by the 35% to 35% missing across quarters. The priced item cell coverage is about 5% across quarters and indicates that cell coverage is sufficient, which will be smoothed across reference quarters comparable to the current CPI composite estimation and ranking process.



Graph 4. Elementary cell coverage percent missing

B. Index estimates

Indexes for the CPI-b, CPI-a, and CPI-q are compared to the CCPI in Graph 5 to demonstrate the impact of the weight revision timelines. As weight representivity improves by decreasing the lag, the smaller the lag more closely approximates the CCPI consistent with results through 2008 from Greenlees and Williams. The terminal index values for index and annual growth rates are summarized in Table 1 along with the size of bias as less the CCPI. The growth rates to September 2021 are higher than through December 2020, but the size of bias patterns remain relatively stable across the fixed quantity versions. The 2021 higher levels of inflation therefore have a small effect on the bias patterns for the CPI-b, CPI-a, and CPI-q.



Graph 5. Index summary of CCPI, CPI-b, CPI-a, and CPI-q (2001=100)

Table 1. Annual growth rate and bias summary (percentage)

	2002-2020	Less CPI		2002-2021'	Less CCPI
CPI-b	2.06	0.24		2.24	0.25
CPI-a	2.03	0.21		2.22	0.22
CPI-q	1.95	0.13		2.14	0.14
CCPI	1.82	-		2.00	-

' = To September 2021 as last published value of CCPI

C. Month-over-month-change and upper-level substitution bias

Upper-level substitution bias is based on the 12-month change to remove in additive effect to indexes over time of the CPI less the CCPI. This difference represents the bias of the CPI fixed quantity weights constraining substitution within the reference period relative the CCPI which reflects substitution in response to price change. For the sum of all months of the absolute value difference the CPI-q reduces upper-level-substitution bias by over 13 percentage points compared to the CPI-b, and by 6.5 percentage points when compared to the CPI-a. For the mean across months for the CPI-q reduces upper level substitution bias by 0.06 compared to the CPI-b, and by 0.03 when compared to the CPI-a. The root mean square error of the absolute value of difference for the CPI-q and CPI-a is 0.02 less than the CPI-b.

Table 2. Upper-level substitution bias summary 2002 to September 2021 (percentage)

	Sum	Mean	Root Mean Square Error
CPI-b	56.66	0.26	0.13
CPI-a	50.03	0.23	0.11
CPI-q	43.57	0.20	0.11

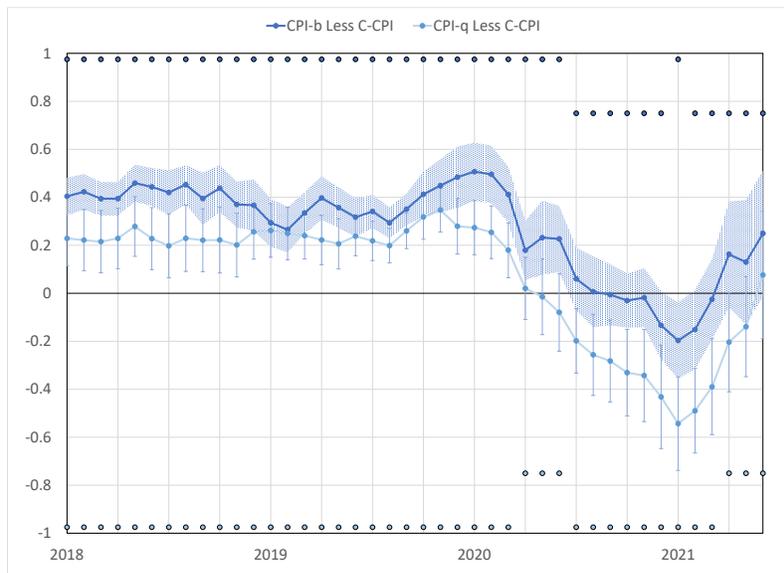
|| = Absolute value of difference

The month-over-month change of index levels is a ratio of the end value to start value less 1. Index levels are sufficient to evaluate the point estimate, but to evaluate Jackknife standard errors and corresponding 95% confidence intervals requires comparing the weighted geographic area as replicates to the all areas as full sample as cost weights for CPI-based formulas. The month-to-month change for the CPI-a and CPI-q and variance formulas are displayed in Appendix B. To calculate the 12-month change of the CPI-q requires processing weights from the current and 3 previous quarters to incorporate quarterly revision chaining. In Graph 6 the January 2018 to June 2021 12-month changes displayed are relative to the left y-axis, and the corresponding Jackknife standard errors are displayed in the right y-axis. The 12-month change for the CCPI is lower than the CPI-b and CPI-q to 1st quarter of 2020 and increases at a faster rate until mid-2nd quarter of 2021. The standard errors range from 0.1 to 0.2 until 1st quarter of 2021, and then they increase to about 0.25 with the corresponding increase to the 12-month changes.



Graph 6. Twelve-month-change and Jackknife standard error: CCPI, CPI-b, CPI-q

The standard errors displayed in Graph 6 can also be processed as a standard error difference to evaluate the corresponding 95% confidence intervals for upper-level substitution bias as described in Appendix B, section III, subsection C (Klick and Shoemaker, 2019). The CPI-b in dark blue and CPI-q in light blue in Graph 7 below that includes significance test as 1 or -1 for the corresponding series. In absolute terms the CPI-q bias is less than the CPI-b to end of 1st quarter 2020. The 2nd quarter 2020 to about 1st quarter the CPI-b bias is less than the CPI-q. And then concludes with the CPI-q bias less than the CPI-b. Herein lies the challenge of evaluating lagged fixed quantity weights revisions in terms of upper-level substitution bias. The CPI-q downward divergences below 0 are concerning, such as for January 2021 and provide a starting point for additional considerations below.



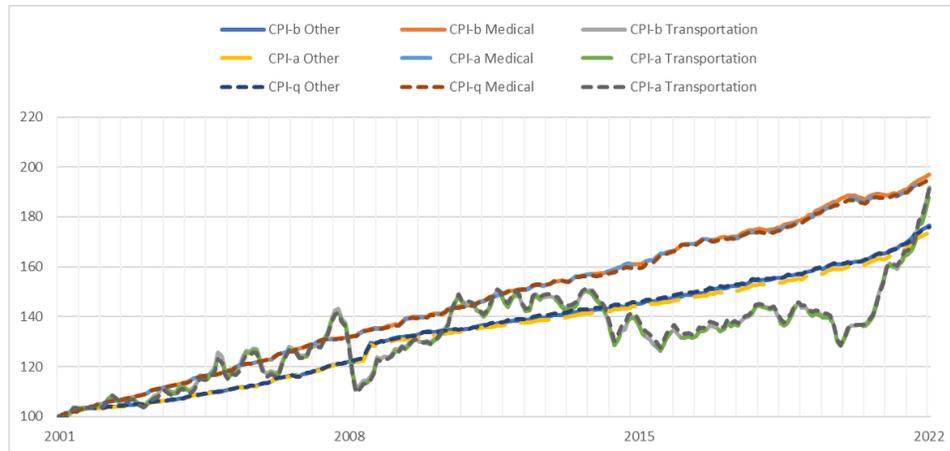
Graph 7. Upper-level substitution bias with Jackknife 95% confidence intervals and significance tests: CPI-b and CPI-q

4. Additional Considerations

A. Sub aggregate indexes

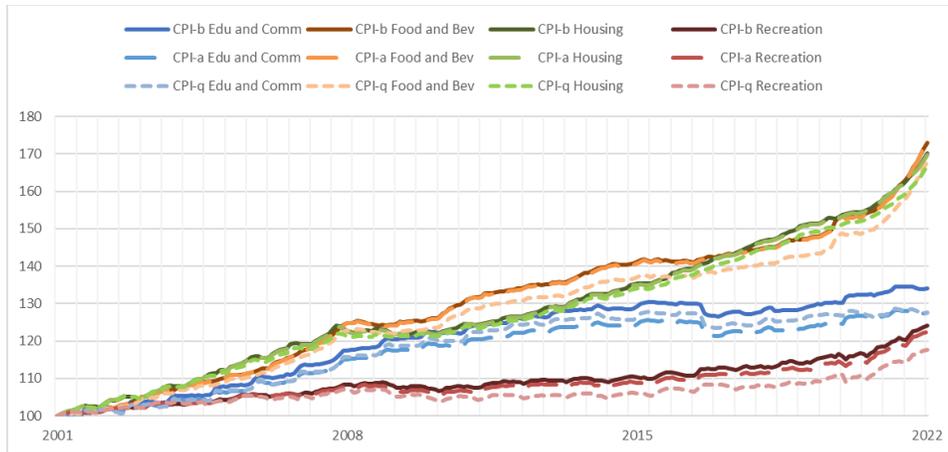
A comparison of the CPI-b, CPI-a, and CPI-q at the major group and region level, the most aggregate level of item and area classification below the all item and all U.S. cities indexes, were used to assess the impacts of quarterly weight revisions to sub aggregate indexes. The results of the geographic analysis, looking at all items for the four Census regions, closely followed the trend of Graph 5, echoing that more frequent revisions improve weight representativity. Geographic analysis can be found in Appendix C.

When looking at major groups for all U.S. cities, the trends were more variable compared to Graph 5 due to the differing changes in prices and consumption for varying items as opposed to an aggregation of all items for all U.S. cities. For example, pricing and consumption of rent, which falls under the housing major group, differ from those of airline fares under the transportation major group. However, the underlying consensus that quarterly weight revisions improve weight representativity largely hold. The CPI-q closely track the CPI-b while remaining smaller than the CPI-b, signaled by an annualized growth rate from 2001 to 2022 of approximately 0.1%, for the following major groups: other goods and services, medical, and transportation.



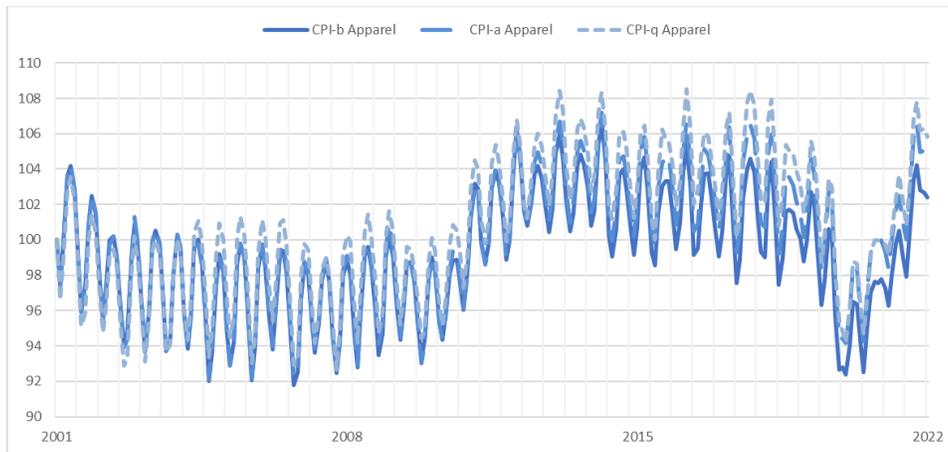
Graph 8. Index summary of CPI-b, CPI-a, and CPI-q, by major group all U.S. cities, where average annual growth rate is approximately 0.1% (2001 = 100)

In Graph 9, the CPI-b is greater than the CPI-q for education and communication, food and beverages, housing, and recreation and their difference in growth rates is greater than 0.1%. This larger difference in growth rates is an indication that quarterly weight revisions may reflect seasonal consumption behavior better than biennial or annual weight revisions for these major groups. Therefore, these major groups may see better weight representativity from utilizing quarterly weights than other goods and services, medical, and transportation.



Graph 9. Index summary of CPI-b, CPI-a, and CPI-q, by major group all U.S cities, where average annual growth rate is greater than 0.1% (2001 = 100)

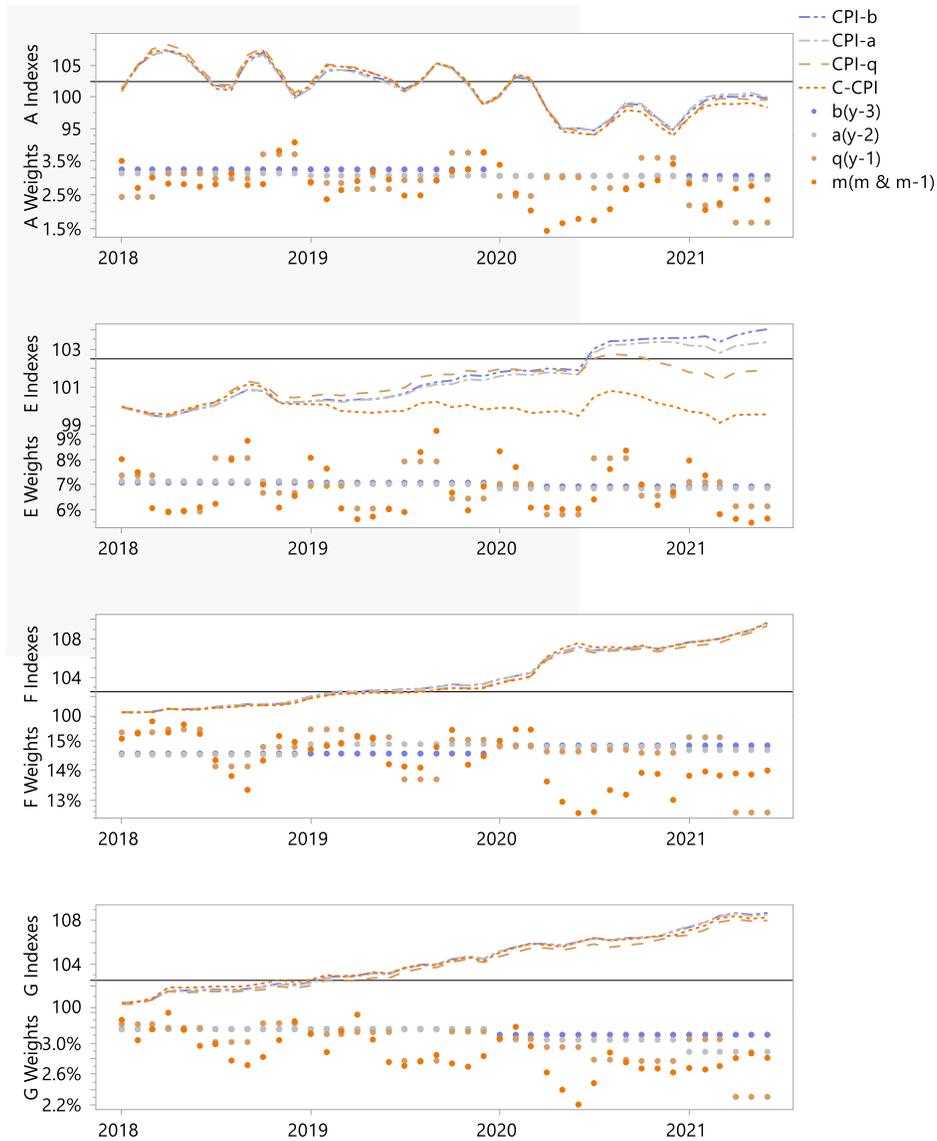
All three weight revision timelines for apparel demonstrate the cyclical seasonality that can occur at the item level. Apparel is the only major group to see the inverse trend of Graph 5, resulting in the CPI-q being greater than the CPI-b as displayed in Graph 10 for an annualized growth rate of less than 0.1%. This introduces the question of chain drift as a result of quarterly revisions, which are more frequent than biennial or annual revisions. Chain drift is the phenomena that occurs when price changes and weight revisions are out of synch causing the index to drift (Cage et. al. 2021).

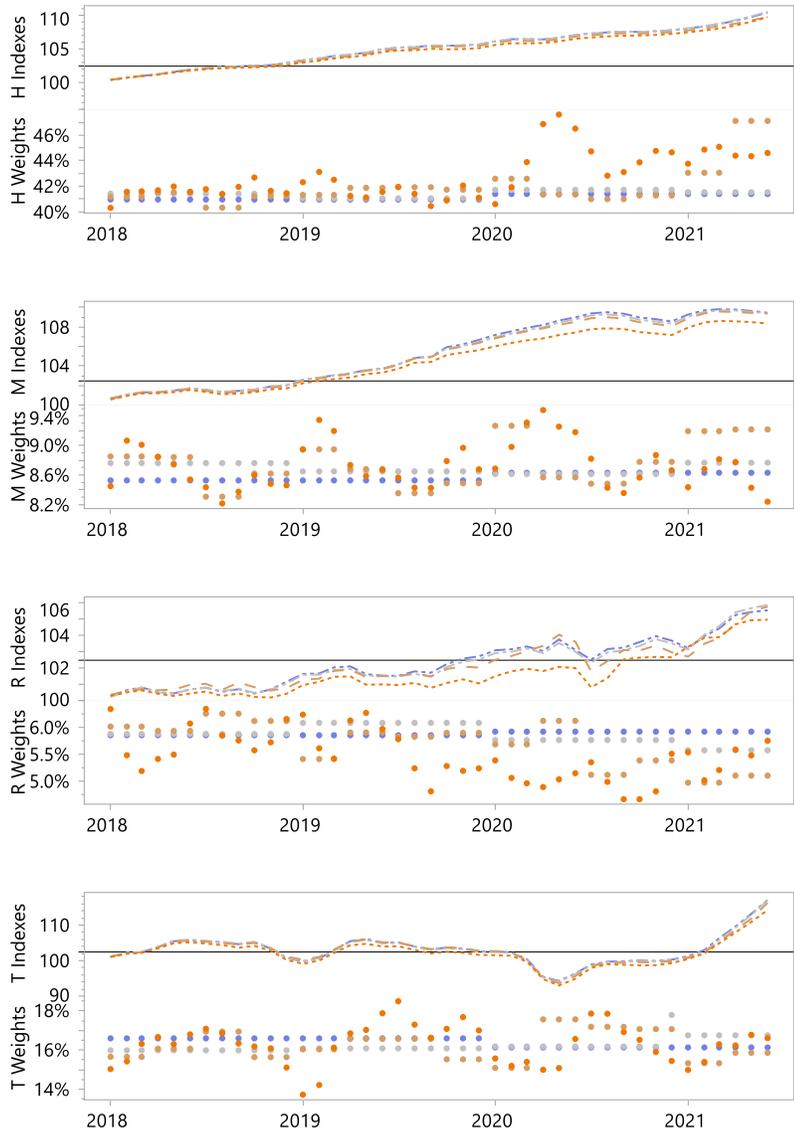


Graph 10. Index summary of CPI-b, CPI-a, and CPI-q, by major group all U.S cities, where average annual growth rate is less than 0.1% (2001 = 100)

To visualize prices changes and weight revisions, a comparison of the indexes for the four weight revision timelines (CPI-b, CPI-a, CPI-q, and CCPI) and their corresponding weights were done between 2018 and 2021. The four indexes, displayed on the top portion, represent price change and trend together for all major groups excluding education and communication. The divergence of indexes in education and communication between the CCPI and CPI-b, CPI-a, and CPI-q may be an indication that price changes being captured by the CCPI is not being reflected in the other weight revision timelines.

The reference period relative importance of the weights used for the indexes, displayed on the bottom portion, showcase consumption patterns for each major group. The relative importance of an item is its expenditure as a percentage of the total expenditure and shows an estimate of how consumers distribute their expenditures amongst various items in the market basket. Some major groups, like apparel and education and communication show more seasonal consumption patterns. Other major groups, such as housing and food and beverages better demonstrate the potential effects of Covid19 pandemic related shutdowns on consumers purchasing behaviors.





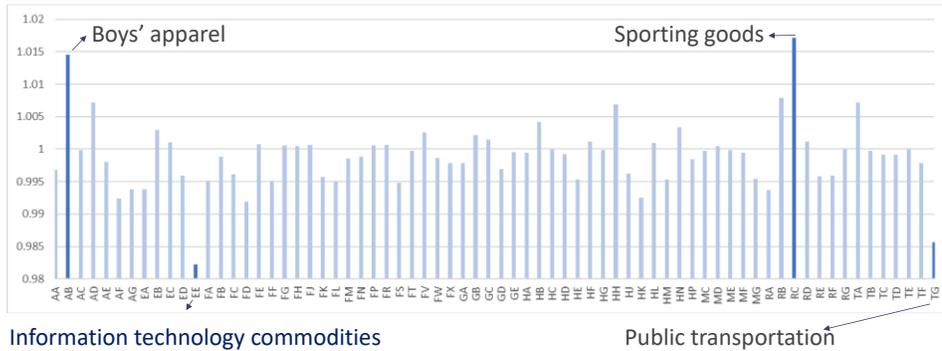
Graph 11. Summary of index and relative importance of weights for CPI-b, CPI-a, and CPI-q, by major group all U.S cities (201712 = 100); Index reference line = 102.5

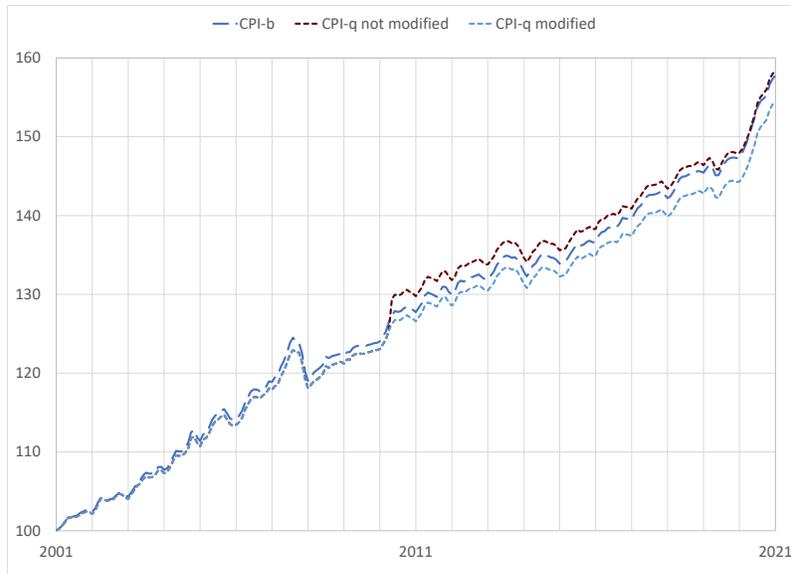
The relative importance of weights for apparel show that biennial weights, denoted by $b(y-3)$, do not reflect the seasonal consumption peaks occurring in December nor the troughs of the first quarter that is seen in the monthly weights, utilized for the CCPI and denoted by $m(m \& m-1)$. Overall, the quarterly weights for all eight major groups better reflect consumption behavior than the biennial weights despite the uncharacteristic shifts occurring in the second quarter of 2020, when the covid-19 pandemic began.

B. Circularity tests of chain drift

The CPI-q above is chained across quarter revisions. A fixed weight version defined as CPI-q* uses the 2017 quarter weights throughout the 2018-2021 4-year index period as a Young formula (International Labour Office 2004, Armknecht 2015). The ratio of growth rates as a circularity test provides a measure of chain drift due to quarter revisions as displayed in Graph 12 (Cage et. al. 2022). The further away from 1 indicates the size of

drift. Expenditure class provides a more detailed item classification than the major group classification above; there are 70 expenditure classes. The circularity test small differences relative to 1 indicate expenditure class q vs. q* weighting differences more so than risk of chain drift due to more frequent quarter revisions. Absolute values of circularity test greater than 0.015 are notable for: Boys apparel (AB), Information technology commodities (EE), Sporting goods (RC), and Public Transportation (TG). Items corresponding to these expenditure classes are displayed in Appendix D where the CPI-q is compared to CPI-b; the long-term biennial revisions proxy the CPI-q*. Individual item indexes use the same prices but diverge due to the q vs. b weight lag and revision frequency.

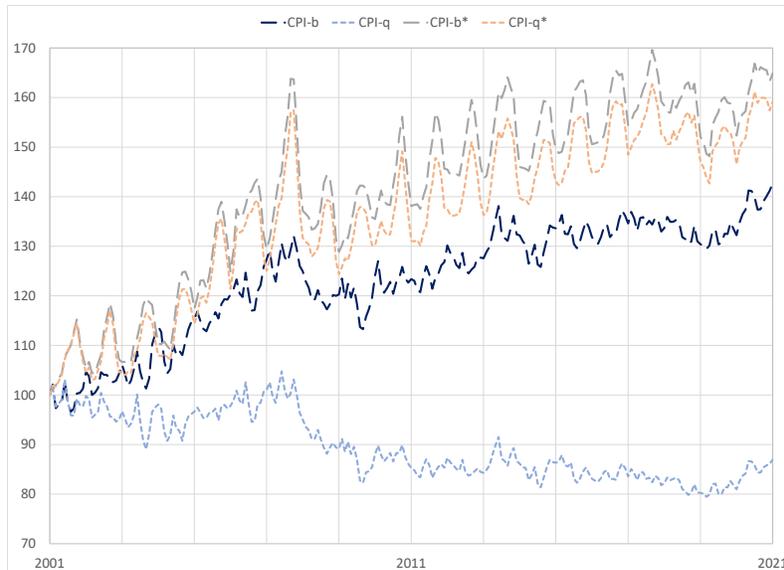




Graph 13. CPI comparison of not modified to modified data for 2011 quarter 2 Financial services for New Jersey-Pennsylvania suburbs

b. Fresh fruits (FK)

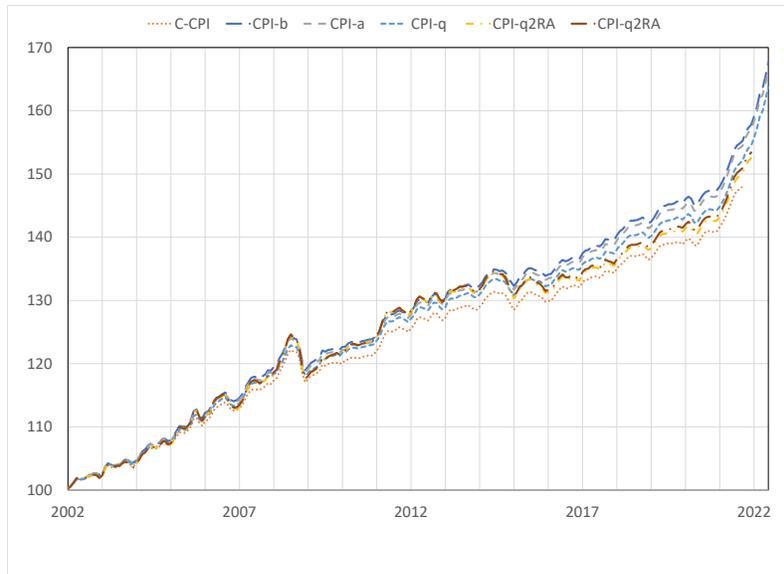
The Fresh Fruits sub aggregate is traditionally subject to risk of drift as displayed in Graph 14 when comparing the CPI-b to the CPI-q. At first glance chain drift appears to be an issue for Fresh Fruits. However, this conclusion is incomplete. When the Other Fresh Fruits basic indexes are excluded as displayed in the CPI-b* and CPI-q*, one of 4 items for this aggregate, the quarterly revisions in orange are like the biennial revisions in gray and appear more seasonal. Other Fresh Fruits is comprised of apricots, avocados, berries, cherries, coconut, grapes, kiwifruit, mangoes, melons, papaya, peaches, pears, pineapples, plums, and pomegranates. Prices and lagged weights for Other Fresh Fruits appears to be contributing disproportionately to the quarterly revised Fresh fruits sub aggregate and will need to be reviewed further to determine how to effectively process the CPI-q for this sub aggregate.



Graph 14. Fresh Fruits CPI-b and CPI-q, and Fresh Fruits less Other Fresh Fruits (FK04)

D. Quarterly averaged weights

When originally drafting the abstract, the authors assumed that quarterly weights represented as rolling averaged reference quarters for a time span of 2 or 4 quarters would reduce the impact of missing expenditure weights at the basic item-area level, and result in indexes nearly as good as the CPI-q when smoothed over quarters, at the expense of minimally increasing the lag as described in Appendix E. Indexes displayed in Graph 15 support this assumption, with limited months where the quarter rolling averages diverge from the CPI-q and CCPI. Analysis of the 12-month changes in Graph 16 contains circles around inflection points where the quarterly averaged indexes diverge from the other series. The size of upper-level substitution bias is summarized as absolute values in Table 3. These divergences potentially indicate that the smoothing over reference quarters is less important than having the most current quarter weight. Future analysis could modify the index reference period used to create the fixed quantity to target reference quarter lagged by 1 year. The potential issue is that the fixed quantity reference period would be defined differently for the weights and indexes and produce unexpected results.



Graph 15. Index comparison to CPI-q2RA and CPI-q4RA



Graph 16. Twelve-month-change comparison

Table 3. Upper-level substitution bias summary

	Sum	Mean	Root Mean Square Error
CPI-b	56.66	0.25	0.13
CPI-a	50.03	0.22	0.11
CPI-q	43.57	0.20	0.11
CPI-q2RA	76.17	0.34	0.28
CPI-q4RA	80.35	0.36	0.28

5. Conclusion

Overall, the above analysis demonstrates that upper-level substitution bias could be reduced via quarterly revisions when compared to biennial and annual revisions. The

downward divergences are concerning such as for January 2021. Graph 7 and additional considerations provide context for the challenge of evaluating weight revisions in terms of upper-level substitution bias. Additionally, this paper describes new methods to calculate quarterly average household expenditures with a month in quarter scope adjustment described in Appendix A, and new formulas described in Appendix B to calculate Jackknife standard errors as 95% confidence intervals for upper-level substitution bias.

The reduction in upper-level substitution bias due to quarterly revisions and improving weight representivity provide motivation to continue and expand research. Quarterly revisions are subject to numerous additional considerations as described above. The review of sub aggregate indexes highlights new considerations for long term and near-term indexes using weights from the corresponding reference period. The risk of chain drift is low, but the Fresh Fruits example demonstrates that Other Fresh Fruits contributing item has a disproportionate effect that will need to be addressed. Similarly, the Financial Services in New Jersey-Pennsylvania suburbs demonstrates that quarter weight revisions will require that reference period index averages be comparable to those of the index period. The analysis of rolling quarter reference periods also demonstrates that the 12-month changes inflection points are subject to a different consideration, possibly due to the rolling quarter reference period average price change representing something different than the superior actual quarter average price change lagged 1 year.

Future research activities include quarter revision analysis of the Wage-earner population, and the preliminary CCPI Constant Elasticity of Substitution formula, both of which are published by the BLS. Time series analysis of weights and corresponding indexes will better explain the seasonal reference period weight effect of quarter revisions. Sub annual quarter revisions are more prone to chain drift than annual based revision, therefore defining a threshold of unacceptable drift will be a useful metric. Finally, reducing the quarter weight lag beyond y-1 to any period less than real time provides limited benefit for seasonal goods and services because lags of 1-3 quarters potentially miss the seasonal trends that occur. Moreover, the upper-level substitution bias results above provide motivation to design a framework for real time household expenditure weights for use by a real time CCPI to remove this bias and most importantly extend analysis in real time.

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Appendix A. Summary of Interview annual and quarterly month in scope adjustment:

1. Annual- 4 reference quarters from 5 collection quarters

		Reference Month												Total	Adjustment	Annual contribution
		1	2	3	4	5	6	7	8	9	10	11	12	36		
Collection year and month	YM1													0	-	0%
	YM2	x												1	1/3	3%
	YM3	x	x											2	2/3	6%
	YM4	x	x	x										3	3/3	8%
	YM5		x	x	x									3	3/3	8%
	YM6			x	x	x								3	3/3	8%
	YM7				x	x	x							3	3/3	8%
	YM8					x	x	x						3	3/3	8%
	YM9						x	x	x					3	3/3	8%
	YM10							x	x	x				3	3/3	8%
	YM11								x	x	x			3	3/3	8%
	YM12									x	x	x		3	3/3	8%
	Y+1M1										x	x	x	3	3/3	8%
	Y+1M2											x	x	2	2/3	6%
	Y+1M3												x	1	1/3	3%

2. Quarterly- 1 reference quarter from 2 collection quarters

		Reference Month			Total	Adjustment	Quarter contribution
		1	2	3	9		
Collection month	1				0	-	0%
	2	x			1	1/3	11%
	3	x	x		2	2/3	22%
	4	x	x	x	3	3/3	33%
	5		x	x	2	2/3	22%
	6			x	1	1/3	11%

Appendix B. Month over month change and Jackknife standard error formulas

I. CPI Lowe formula annual based revisions

A. Cost Weight for urban population: $CW(A, I, f, t) = IX(A, I, f, t) \times AGGWT(A, I, f, t)$

CW = Cost Weight

IX = Index

AGGWT = Aggregation weight is contributing weight of basic index

A = Set of area strata (US city average)

I = Set of item strata (All-Items)

f = Full sample

t = Index month

B. Full sample 12-month percent change US level:

For odd years: $PC(A, I, f, t, t - 12) = \left(\frac{CW(A, I, f, t)}{CW(A, I, f, t-12)} - 1 \right) \times 100$

For even years (bridge over 1 revision):

$$PC(A, I, f, t, t - 12) = \left(\frac{CW(A, I, f, t)}{CW(A, I, f, t - 12)} \times \frac{CW(A, I, f, v, OLD)}{CW(A, I, f, v, NEW)} - 1 \right) \times 100$$

PC = 12 month Percent Change

t-12 = 12 months prior

v = Pivot month precedes the first index month of weight revision (December)

OLD = Previous biennial period aggregation weight

NEW = Current biennial period aggregation weight

C. Variance jackknife method of 12-month percent change: full sample (f) less replicate level (r):

Replicate odd years: $PC(A - a, I, r, t, t - 12) = \left(\frac{CW(A, I, f, t) - CW(a, I, r, t)}{CW(A, I, f, t-12) - CW(a, I, r, t-12)} - 1 \right) \times 100$

Replicate even years:

$$PC(A - a, I, r, t, t - 12) = \left(\frac{\left[CW(A, I, f, t) \times \frac{CW(A, I, f, v, OLD)}{CW(A, I, f, v, NEW)} \right] - \left[CW(a, I, r, t) \times \frac{CW(a, I, r, v, OLD)}{CW(a, I, r, v, NEW)} \right]}{CW(A, I, f, t - 12) - CW(a, I, r, t - 12)} - 1 \right) \times 100$$

a= Individual area strata

r = Replicate

Variance of 12-month percent change:

$$V[PC(A, I, f, t, t - 12)] = \left(\frac{N-1}{N}\right) \sum_{r=1}^N [PC(A - a, I, r, t, t - 12) - PC(A, f, t, t - 12)]^2$$

N= 32 strata for 2010 geography effective 2018 indexes forward, 38 1990 geography for indexes 1999-2017

$$SE[PC(A, I, f, t, t - 12)] = \sqrt{V[PC(A, I, f, t, t - 12)]}$$

D. Variance jackknife method of 12-month change difference of subgroup population or formulas (SUB) to reference product(P):¹

Variance of 12-month percent change difference:

$$V[PC(SUB: P, A, I, f, t, t - 12)] = \left(\frac{N-1}{N}\right) \sum_{r=1}^N ([PC(SUB, A - a, I, r, t, t - 12) - PC(SUB, A, f, t, t - 12)] - [PC(P, A - a, I, r, t, t - 12) - PC(P, A, f, t, t - 12)])^2$$

$$\text{Standard error difference: } SE[PC(SUB: P, A, I, f, t, t - 12)] = \sqrt{V[PC(SUB: P, A, I, f, t, t - 12)]}$$

$$\text{Z-Score: } z[PC(SUB: P, A, I, f, t, t - 12)] = \frac{[PC(SUB: P, A, I, f, t, t - 12)] - 0}{SE[PC(SUB: P, A, I, f, t, t - 12)]}$$

$$\text{P-Value: } pval[PC(SUB: P, A, I, f, t, t - 12)] = 2 \times (1 - probnorm(z[PC(SUB: P, A, I, f, t, t - 12)]))$$

95% Confidence Interval:

$$\begin{aligned} 95\%CI[PC(SUB: P, A, I, f, t, t - 12)] \\ = \mu[PC(SUB: P, A, I, f, t, t - 12)] \\ \pm (SE[PC(SUB: P, A, I, f, t, t - 12)] \times 1.96) \end{aligned}$$

Underlying Correlation: $Rho[PC(SUB: P, A, I, f, t, t - 12)] =$

$$\frac{\left(V[PC(SUB: P, A, I, f, t, t - 12)]^2 - \left(\begin{array}{l} SE[PC(SUB, A, I, f, t, t - 12)]^2 \\ - SE[PC(P, A, I, f, t, t - 12)]^2 \end{array} \right) \right)}{(2 \times SE[PC(SUB, A, I, f, t, t - 12)] \times SE[PC(P, A, I, f, t, t - 12)])}$$

II. CPI Lowe formula quarter based revisions

A. Cost Weight urban population for All Items: $CW(A, I, f, q, t) = IX(A, I, f, t) \times AGGWT(A, I, f, q)$

q= Quarter expenditure weights for index month lagged by 1 year due to processing

¹ Difference operator based on part on Shoemaker 2017, and Internal BLS Variance Monitor, although these references use SRG method for variance estimates.

B. Full sample 1-month percent change US level:

1. Within quarter revision (2nd and 3rd months): $PC1M(A, I, t, q, t - 1) = \left(\frac{CW(A, I, q, t)}{CW(A, I, q, t-1)} - 1 \right) \times 100$

2. Bridge across quarter revision:

$$PC1M(A, I, q, t, t - 1) = \left(\frac{CW(A, I, q, t)}{CW(A, I, q, t - 1)} \times \frac{CW(A, I, qv, q - 1)}{CW(A, I, qv, q)} - 1 \right) \times 100$$

PC1M = Percent change 1 month

qv = Quarter pivot month precedes first month of quarterly revision (December, March, June, September)

t-1 = 1 month prior

q-1 = Previous quarter expenditure weights

C. Full Sample 12-month percent change bridge across 4 quarterly revisions

$$PC(A, I, q, t, t - 12) =$$

$$\left(\left(\frac{CW(A, I, f, t, q)}{CW(A, I, f, t - 12, q - 4)} \times \frac{CW(A, I, f, qv, q - 1)}{CW(A, I, f, qv, q)} \times \frac{CW(A, I, f, qv_{q-1}, q - 2)}{CW(A, I, f, qv_{q-1}, q - 1)} \times \frac{CW(A, I, f, qv_{q-2}, q - 3)}{CW(A, I, f, qv_{q-2}, q - 2)} \times \frac{CW(A, I, f, qv_{q-3}, q - 4)}{CW(A, I, f, qv_{q-3}, q - 3)} \right) - 1 \right) \times 100$$

qv_{q-1} = Quarter pivot month occurs 1 quarter prior t

q-2 = 2 quarters prior expenditure weights

qv_{q-2} = Quarter pivot month occurs 2 quarters prior t

q-3 = 3 quarters prior expenditure weights

qv_{q-3} = Quarter pivot month occurs 3 quarters prior t

q-4 = 4 quarters prior expenditure weights

D. Variance jackknife method of 12-month percent change bridge across 4 quarterly revisions:

Replicate:

$$PC(A - a, I, r, q, t, t - 12)$$

$$\left(\left(\frac{CW(A, I, f, t, q)}{CW(A, I, f, t - 12, q - 4)} \times \frac{CW(A, I, f, qv, q - 1)}{CW(A, I, f, qv, q)} \times \frac{CW(A, I, f, qv_{q-1}, q - 2)}{CW(A, I, f, qv_{q-1}, q - 1)} \times \frac{CW(A, I, f, qv_{q-2}, q - 3)}{CW(A, I, f, qv_{q-2}, q - 2)} \times \frac{CW(A, I, f, qv_{q-3}, q - 4)}{CW(A, I, f, qv_{q-3}, q - 3)} \right) - \left(\frac{CW(a, I, r, t, q)}{CW(a, I, r, t - 12, q - 4)} \times \frac{CW(a, I, r, qv, q - 1)}{CW(a, I, r, qv, q)} \times \frac{CW(a, I, r, qv_{q-1}, q - 2)}{CW(a, I, r, qv_{q-1}, q - 1)} \times \frac{CW(a, I, r, qv_{q-2}, q - 3)}{CW(a, I, r, qv_{q-2}, q - 2)} \times \frac{CW(a, I, r, qv_{q-3}, q - 4)}{CW(a, I, r, qv_{q-3}, q - 3)} \right) \right) \times 100$$

Variance of 12-month percent change across 4 quarterly revisions:

$$V[PC(A, I, f, q, t, t - 12)] = \left(\frac{N - 1}{N} \right) \sum_{r=1}^N [PC(A - a, I, r, q, t, t - 12) - PC(A, I, f, q, t, t - 12)]^2$$

$$SE[PC(A, I, f, q, t, t - 12)] = \sqrt{V[PC(A, I, f, q, t, t - 12)]}$$

E. Variance jackknife method of 12-month change difference of subgroup population or formulas (SUB) to reference product(P) across 4 quarterly revisions:

Variance of 12-month percent change difference:

$$V[PC(SUB: P, A, I, f, q, t, t - 12)] = \left(\frac{N - 1}{N} \right) \sum_{r=1}^N ([PC(SUB, A - a, I, r, q, t, t - 12) - PC(SUB, A, f, q, t, t - 12)] - [PC(P, A - a, I, r, q, t, t - 12) - PC(P, A, f, q, t, t - 12)])^2$$

$$\text{Standard error difference: } SE[PC(SUB: P, A, I, f, q, t, t - 12)] = \sqrt{V[PC(SUB: P, A, I, f, q, t, t - 12)]}$$

$$\text{Z-Score: } z[PC(SUB: P, A, I, f, q, t, t - 12)] = \frac{[PC(SUB: P, A, I, f, q, t, t - 12)] - 0}{SE[PC(SUB: P, A, I, f, q, t, t - 12)]}$$

P-Value: $pval[PC(SUB: P, A, I, f, q, t, t - 12)] = 2 \times (1 - probnorm(z[PC(SUB: P, A, I, f, q, t, t - 12)]))$

95% Confidence Interval:

$$\begin{aligned} &95\%CI[PC(SUB: P, A, I, f, q, t, t - 12)] \\ &= [PC(SUB: P, A, I, f, q, t, t - 12)] \\ &\pm (SE[PC(SUB: P, A, I, f, q, t, t - 12)] \times 1.96) \end{aligned}$$

Underlying Correlation:

$$\begin{aligned} &Rho[PC(SUB: P, A, I, f, q, t, t - 12)] = \\ &\frac{\left(V[PC(SUB: P, A, I, f, q, t, t - 12)]^2 - \left(\begin{array}{l} SE[PC(SUB, A, I, f, q, t, t - 12)]^2 \\ -SE[PC(P, A, I, f, q, t, t - 12)]^2 \end{array} \right) \right)}{(2 \times SE[PC(SUB, A, I, f, q, t, t - 12)] \times SE[PC(P, A, I, f, q, t, t - 12)])} \end{aligned}$$

III. Final CCPI compared to CPI biennial and quarterly revisions

The price change of urban population final CCPI uses a Tornqvist formula as a geometric average weighted as a 2 month moving average of monthly weights that reflect consumption and substitution concurrent with the index month. Analysis of the 12-month change across the final CCPI and CPI formulas uses index levels.

A. Full sample 12-month percent change US level:

1. Final CCPI (FC):

$$\text{Price relative 1 month: } PR_{FC,IA,f,t-1,t} = \prod_{I,A} \left(\frac{IX_{i,a,t}}{IX_{i,a,t-1}} \right)^{\left(\frac{S_{i,a,t} + S_{i,a,t-1}}{2} \right)}$$

S= share month monthly weights

$$\begin{aligned} \text{Price change 12 months: } &PC(FC, A, I, f, t, t - 12) = \\ &(\prod PR_{FC,IA,f,t-12,t} - 1) \times 100 \end{aligned}$$

2. CPI-b: $PC(b, A, I, f, t, t - 12) =$ See I.B.

3. CPI-q: $PC(q, A, I, f, t, t - 12) =$ See II.C.

Final CCPI:

B. Variance jackknife method of 12-month percent change: full sample (f) less replicate level (r):

1. Final CCPI

a. Replicate:

Relative:

$$PR_{FC,IA-a,f,t-1,t} = \prod_{I,A-a} \left(\frac{IX_{i,a,t}}{IX_{i,a,t-1}} \right)^{\left(\frac{S'_{i,a,t} + S'_{i,a,t-1}}{2} \right)}$$

S'= share month monthly weights where denominator is A-a.

Price change 12 months:

$$PC(FC, A - a, I, r, t, t - 12) = (\prod PR_{FC,IA-a,f,t-12,t} - 1) \times$$

100

b. Variance

$$\begin{aligned}
& V[PC(FC, A, I, f, t, t - 12)] \\
&= \left(\frac{N - 1}{N}\right) \sum_{r=1}^N (PC(FC, A - a, I, r, t, t - 12) \\
&\quad - PC(FC, I, A, f, t, t - 12))^2
\end{aligned}$$

$$c. SE[PC(FC, A, I, f, t, t - 12)] = \sqrt{V[PC(FC, A, I, f, t, t - 12)]}$$

2. CPI-b: $PC(b, A, I, f, t, t - 12)$ = See I.C
3. CPI-q: $PC(q, A, I, f, t, t - 12)$ = See II.D

C. Variance jackknife method of 12-month change difference of CPI-b or CPI-q revision version (RV) to final CCPI (FC):

Variance of 12-month percent change difference:

$$\begin{aligned}
& V[PC(RV: FC, A, I, f, t, t - 12)] \\
&= \left(\frac{N - 1}{N}\right) \sum_{r=1}^N ([PC(RV, A - a, I, r, t, t - 12) - PC(RV, A, f, t, t - 12)] \\
&\quad - [PC(FC, A - a, I, r, t, t - 12) - PC(FC, A, f, t, t - 12)])^2
\end{aligned}$$

$$\begin{aligned}
& \text{Standard error difference: } SE[PC(RV: FC, A, I, f, t, t - 12)] = \\
& \sqrt{V[PC(RV: FC, A, I, f, t, t - 12)]}
\end{aligned}$$

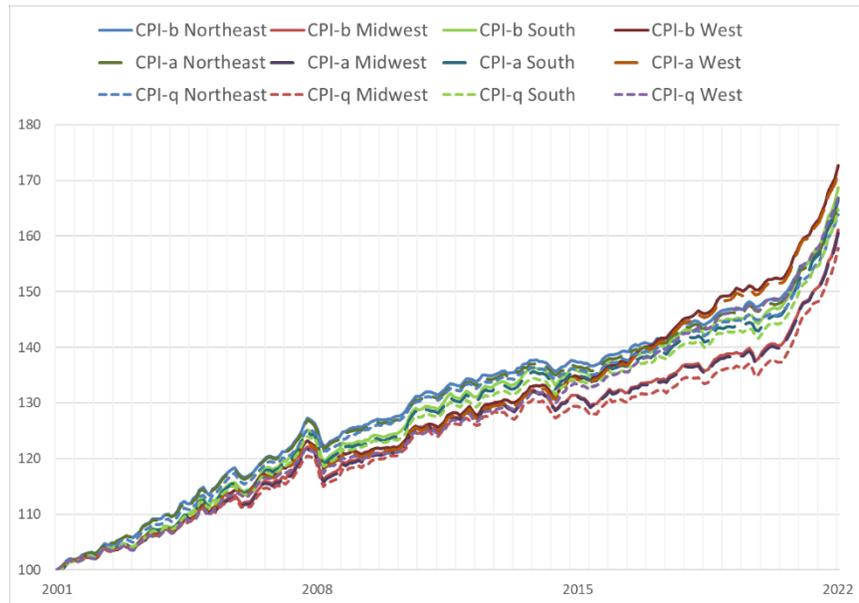
$$\begin{aligned}
& \text{Z-Score: } z[PC(RV: FC, A, I, f, t, t - 12)] = \\
& \frac{|[PC(RV: FC, A, I, f, t, t - 12)] - 0|}{SE[PC(RV: FC, A, I, f, t, t - 12)]}
\end{aligned}$$

$$\begin{aligned}
& \text{P-Value: } pval[PC(RV: FC, A, I, f, t, t - 12)] = 2 \times (1 - \\
& probnorm(z[PC(RV: FC, A, I, f, t, t - 12)]))
\end{aligned}$$

95% Confidence Interval:

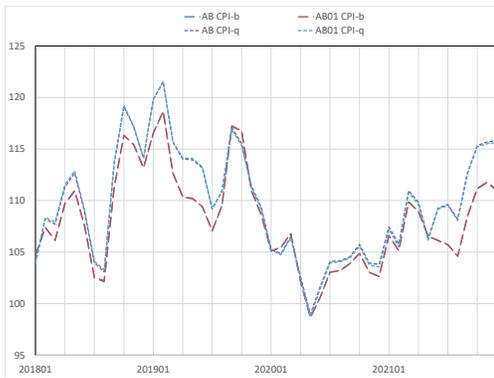
$$\begin{aligned}
& 95\%CI[PC(RV: FC, A, I, f, t, t - 12)] \\
&= PC(RV: FC, A, I, f, t, t - 12) \\
&\pm (SE[PC(RV: FC, A, I, f, t, t - 12)] \times 1.96)
\end{aligned}$$

Appendix C. Index summary of CPI-b, CPI-a, and CPI-q, all items by region (200112 = 100)

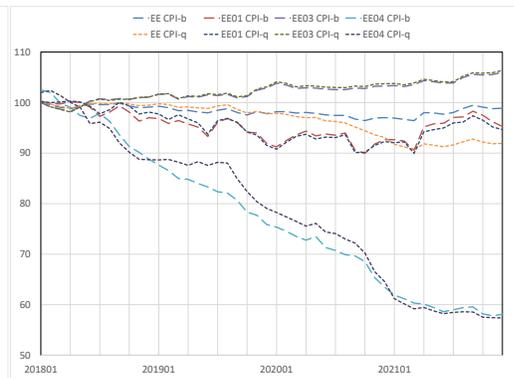


Appendix D. Circularity test of CPI-b compared to CPI-q 2018-May 2021 for subset of expenditure class codes, 201712 = 100

The January 2018 to May 2022 CPI-b and CPI-q are displayed for Graphs x-y. Weighting the basic level cells as b vs. q results in indexes diverging and may even result in the aggregate moving in an opposite direction. For Boys Apparel, the May CPI-b less CPI-q is -5 points, comparable to the MG results above. For Information and technology commodities, the May CPI-b less CPI-q is 7 points due to Telephone hardware, calculators, and other consumer information items' (EE04) large CPI-q move downward representing about 9% of the expenditure class. For Sporting goods, the May CPI-b less CPI-q is -8 points due to Sports vehicles including bicycles' (RC01) large CPI-q move upward representing about 50% of the expenditure class. And for Public transportation, the May CPI-b less CPI-q is 3 points due to Other intercity transportation (TG02), which represents about 14% of the expenditure class. The CPI-b vs. CPI-q divergences can be explained by individual item indexes, which could be explored in more detail at the basic item area level, and therefore the risk of chain drift due to quarterly revisions is low.



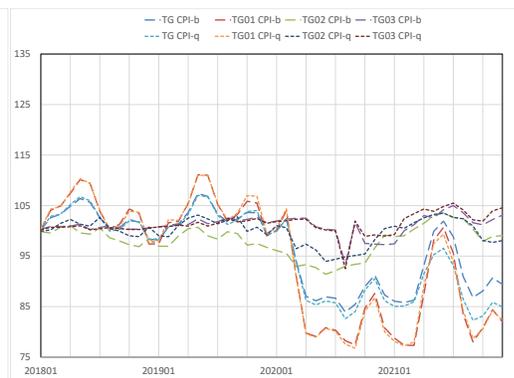
Graph Boys Apparel (AB)



Graph Information technology commodities (EE)



Graph Sporting goods (RC)



Graph Public Transportation (TG)

Appendix E. Weight revision timelines for 2 and 4 quarter rolling averages

For the 2-quarter rolling average weight revision, the indexes for the first quarter of 2022 use weights from 2020 quarter 4 to 2021 quarter 1 as its reference period. This results in an average 13.5-month lag. Under this revision timeline, the indexes for the second quarter of 2022 would use quarterly weights from the first and second quarter of 2021 while the pivot month would move from December 2021 to March 2022.



Graph 2-quarter rolling average

For 4-quarter rolling averages, the indexes for the first quarter of 2022 use weights from the second, third and fourth quarter of 2020 and the first quarter of 2021. This results in an average 16.5-month lag. Under this revision timeline, the indexes for the second quarter of 2022 use the second half of the weights from 2020 and the first half of 2021.



Graph 4-quarter rolling average