

Testing stochastic rationality and normality of demand with unobserved heterogeneity and endogenous expenditures: the case of two goods

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- ▶ **Normality of goods** improve predicting power of fully nonparametric estimation approach.

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- ▶ **Not restrictive.** Multi-goods setting can be reduced to a two-goods setting using Hicksian aggregation or weakly separability for dimensionality reduction.
- ▶ An empirical illustration to data drawn from the U.S. Consumer Expenditure Survey ([1994-2007](#)).

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- ▶ **Cherchye, De Rock and Demuyne (2018):** Normality of demand in a two-goods setting. Dataset $S = \{p_t, q_t, x_t\}_{t \in T}$, is consistent with weak normality of goods q and z if and only if for all $t, s \in T$:

$$\text{if } p_t < p_s \text{ and } q_t \leq q_s \text{ then } z_t \leq z_s$$

Testing Results

Normality of Goods q and z : If the cross-sectional joint distribution of $\{q_{t,j}, z_{t,j}, x_{t,j}\}_{t \in T}$ on budgets $\{p_t\}_{t \in T}$ are stochastically consistent with normality of both goods q and z , then for every $t, s \in T$ such that $p_t < p_s$, it must be that

$$Pr[q_{t,j} \leq q_{s,j} \text{ and } z_{t,j} \geq z_{s,j}] = 0.$$

Then, for any $a, b \in \mathbb{R}_+$,

$$Pr[q_{t,j} \leq a \text{ and } z_{t,j} \geq b] + Pr[a \leq q_{s,j} \text{ and } b \geq z_{s,j}] \leq 1.$$

Prediction results

Bounds on $Pr[q_j(p_0, x_0) \leq q_0]$ assuming goods q and z are both normal

$$\max_{p_t < p_0} Pr[q_{t,j} \leq q_0 \text{ and } z_{t,j} \geq z_0] \leq Pr[q_j(p_0, x_0) \leq q_0]$$

$$Pr[q_j(p_0, x_0) \leq q_0] \geq \min_{p_t > p_0} Pr[q_{t,j} \leq q_0 \text{ or } z_{t,j} \geq z_0]$$

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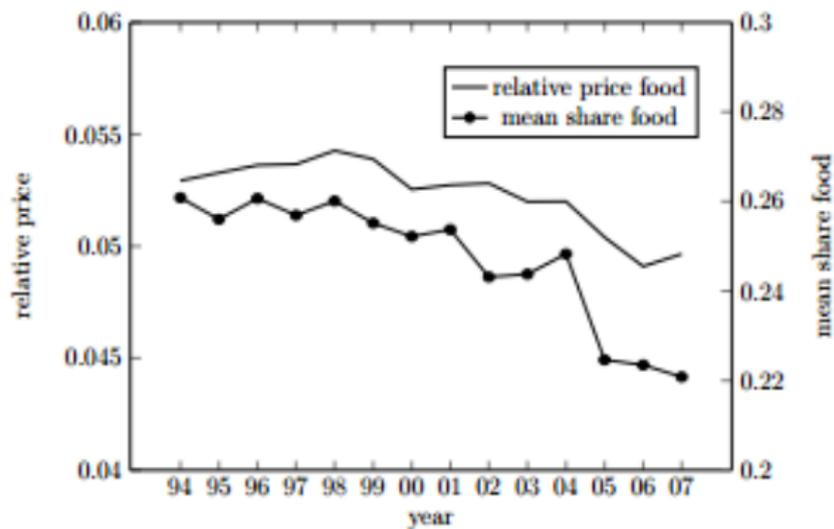
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- ▶ By conditioning on housing status and vehicle ownership, the impact of the implicit assumption of separability between durable and non-durable consumption is mitigated.

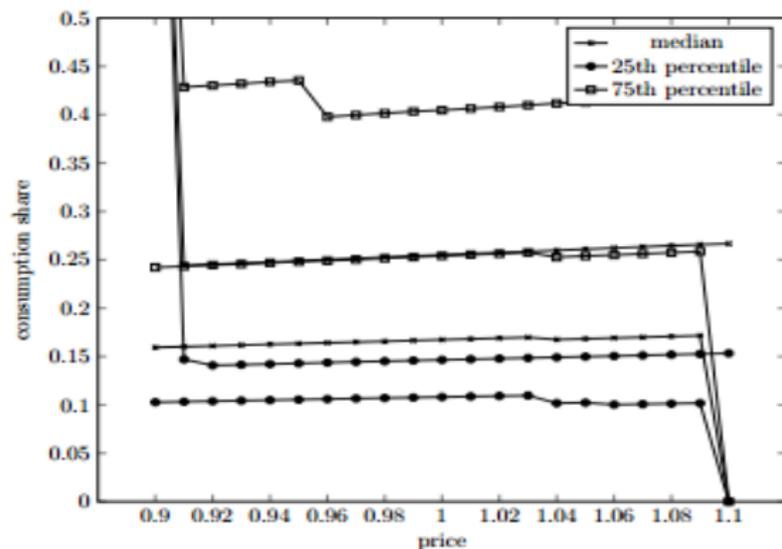
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Figure 4: Evolution of relative price and consumption share of food



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Figure 5: Quantiles bounds for $[\eta_\ell, \eta_h] = [0, 1]$ (food as necessary good)



THANK YOU!