1. **Price Index Formula Notation**

- **Abbreviation of underlying price index formula**
- **Index symbol**
- **Item, area, and population represented**
- **Base period of index**
- **Comparison period of index**

2. **The Cost-of-Living Index Concept**

\[
iX^C_{[0,t]} = \frac{\min \sum_i i \cdot p \times q_i}{\sum_i i \cdot p_0 \times q_0}
\]

- The minimum expenditure \((P_t \times Q_t)\) required in comparison period \((t)\) to attain the same level of satisfaction or utility \((U_0)\) achieved in base period \((0)\), divided by the actual expenditure \((P_0 \times Q_0)\) in base period \((0)\).

3. **Price Index Formulas Commonly Used to Approximate a Cost-of-Living Index**

**FIRST ORDER APPROXIMATIONS:**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>LASPEYRES</td>
<td>(IX^L_{[0,t]} = \sum_i \frac{i \cdot p_t}{p_0} )</td>
</tr>
<tr>
<td>PAASCHE</td>
<td>(IX^P_{[0,t]} = \left[ \sum_i \frac{i \cdot p_t}{p_0} \right]^{-1} )</td>
</tr>
<tr>
<td>GEOMETRIC MEAN</td>
<td>(IX^G_{[0,t]} = \prod_i \left( \frac{i \cdot p_t}{i \cdot p_0} \right)^{x_t} )</td>
</tr>
</tbody>
</table>

**SECOND ORDER APPROXIMATIONS:**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>TORNQVIST</td>
<td>(IX^T_{[0,t]} = \prod_i \left( \frac{i \cdot p_t}{i \cdot p_0} \right)^{x_t} )</td>
</tr>
<tr>
<td>FISHER IDEAL</td>
<td>(IX^F_{[0,t]} = \left( (IX^L_{[0,t]} \times IX^P_{[0,t]}) \right)^{1/2} )</td>
</tr>
</tbody>
</table>

**KEY:**

- \(i \cdot p_t\) = Price of item \((i)\) in comparison period \((t)\)
- \(i \cdot p_0\) = Price of item \((i)\) in base period \((0)\)
- \(i \cdot x_t\) = Expenditure on item \((i)\) in comparison period \((t)\), divided by expenditures on all items in comparison period \((t)\)
- \(i \cdot x_0\) = Expenditure on item \((i)\) in base period \((0)\), divided by expenditures on all items in base period \((0)\)
4. Estimation of Price Change in the Chained Consumer Price Index (C-CPI-U)

**LOWER-LEVEL AGGREGATION:**

\[
i_{a} \, I_{X}^{L}[0,t] = \sum_{k \in i,a} k \, S_{0} \left( \frac{k \, p_{t}}{k \, p_{0}} \right)
\]

or

\[
i_{a} \, I_{X}^{G}[0,t] = \prod_{k \in i,a} \left( \frac{k \, p_{t}}{k \, p_{0}} \right) k \, S_{0}
\]

**UPPER-LEVEL AGGREGATION:**

<table>
<thead>
<tr>
<th></th>
<th>Long-term Price Change</th>
<th>Month-to-Month Price Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial C-CPI-U</strong></td>
<td>( I_{A} , I_{X}^{G_{i}}[z; y, t] = I_{A} , I_{X}^{G_{r}}[z; y-1, 12] \times \prod_{n=1}^{t} I_{A} , I_{X}^{G_{i}}[n-1; n] )</td>
<td>( I_{A} , I_{X}^{G}[t-1; t] = \lambda_{y} \prod_{i,a \in I,A} \left( \frac{i_{a} , I_{X}^{L_{o}G_{b}}[0; t]}{i_{a} , I_{X}^{L_{o}G_{b}}[0; t-1]} \right) i_{a} S_{b} )</td>
</tr>
<tr>
<td><strong>Interim C-CPI-U</strong></td>
<td>( I_{A} , I_{X}^{G_{i}}[z; y, t] = I_{A} , I_{X}^{T}[z; y-1, 12] \times \prod_{n=1}^{t} I_{A} , I_{X}^{G_{i}}[n-1; n] )</td>
<td>( I_{A} , I_{X}^{G}[t-1; t] = \lambda_{y} \prod_{i,a \in I,A} \left( \frac{i_{a} , I_{X}^{L_{o}G_{b}}[0; t]}{i_{a} , I_{X}^{L_{o}G_{b}}[0; t-1]} \right) i_{a} S_{b} )</td>
</tr>
<tr>
<td><strong>Final C-CPI-U</strong></td>
<td>( I_{A} , I_{X}^{L_{o}T}[z; y, t] )</td>
<td>( I_{A} , I_{X}^{T}[t-1; t] = \prod_{i,a \in I,A} \left( \frac{i_{a} , I_{X}^{L_{o}G_{b}}[0; t]}{i_{a} , I_{X}^{L_{o}G_{b}}[0; t-1]} \right) i_{a} S_{b} )</td>
</tr>
</tbody>
</table>

**KEY:**

- \( k \) = unique good or service
- \( A \) = CPI aggregate area
- \( a \) = CPI elementary area
- \( i \) = CPI elementary item
- \( 0 \) = elementary index base period
- \( t \) = month
- \( y \) = year
- \( p_{t} \) = price of good \((k)\) in month \((t)\)
- \( p_{0} \) = price of good \((k)\) in base-period \((0)\)
- \( x_{o} \) = expenditure for good \((k)\) in base period \((0)\), divided by expenditure for all \((k)\) goods in elementary item \((i)\), area \((a)\) in base period \((0)\)
- \( z \) = December 1999 index base period
- \( b_{i} \) = expenditure reference period of CPI-U index of year \((y)\);
  NOTE: \( b_{i} = 1999-2000 \) for \( y = 2002 \) and \( y = 2003 \).
- \( b_{i} \) = expenditure reference period of CPI-U index of year \((y+1)\);
  NOTE: \( b_{i} = b_{i-1} \) for \( y = 2002 \) and \( b_{i} = 2001-2002 \) for \( y = 2003 \).
- \( \lambda_{y} \) = Adjustment factor used in year \((y)\) to calculate Initial \((y)\) and Interim \((y-1)\) C-CPI-U indexes published in year \((y)\); NOTE: \( \lambda_{y} = 1 \) for C-CPI-U indexes published in 2002.

\( I_{X}^{L} \) = Laspeyres elementary index
\( I_{X}^{G} \) = Geometric Mean elementary index
\( I_{X}^{G_{i}} \) = Initial C-CPI-U index
\( I_{X}^{T} \) = Interim C-CPI-U index
\( I_{X}^{T} \) = Final C-CPI-U index
\( i_{a} S_{b} \) = expenditure for elementary item \((i)\) in area \((a)\) in expenditure period \((b)\), divided by expenditure for all elementary items in aggregate item \((I)\) in aggregate area \((A)\) in expenditure period \((b)\)
\( i_{a} S_{b} \) = expenditure for elementary item \((i)\) in area \((a)\) in expenditure period \((b)\), divided by expenditure for all elementary items in aggregate item \((I)\) in aggregate area \((A)\) in expenditure period \((b)\)
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