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The Problem with Normalizing Preferences that Change in a Cost-of-Living Index

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Abstract

There are new proposals for prices indexes that attempt to correct for what they consider bias in standard indexes from changes in consumer preferences. But these proposals have a fundamental problem that changes in preferences between two periods cannot be identified by data on prices and quantities with only a normalization. This paper shows that the required normalization is not free, so that an arbitrary choice of normalization can yield any desired index result. In fact, a normalization using the Sato-Vartia weights yields a Sato-Vartia index, implying exactly zero bias.

1 Introduction

Multiple prices indexes that incorporate changing consumer preferences have been proposed. Such studies claim that standard cost-of-living indexes (COLIs) suffer from bias because they are defined for only one set of preferences. These studies include Redding & Weinstein (2020) (RW), Zadrozny (2023), Gábor-Tóth & Vermeulen (2022), Abe, Inoue, and Sato (2022), Abe and Rao (2020), and Hottman & Monarch (2020; 2023), and Erlich, et al. (2022) also studies this issue. But these proposals have a fundamental problem that changes in preferences between two periods cannot be identified by data on prices and quantities. At any given time, the data can only identify relative preferences over goods in that time period, not how the level of the standard of living (utility) compares between periods. Therefore, they use a normalization to fix the level. Using the RW framework as an example, this paper shows that the required normalization is not free, so that an arbitrary choice among different normalizations will yield almost any desired index result.[1]

[1]Abe and Rao (2020) show (in Result 7) that any normalization of this type must be a weighted geometric mean for the index to have the property of commensurability, which

For illustration, consider if there were only two goods, apples and oranges, in two periods. If consumers simply enjoy both proportionately more in the second period, so that consuming the same quantities gives a higher level of the standard of living, the cost of obtaining the standard of living of the first period in the second period would be lower and the COLI would fall. There would be no way to measure this change with price and quantity data because the observed shares would be the same. Therefore, these proposals normalize preferences to prevent an overall rise or fall by dividing the parameters by some function of them, such as the mean of the desire for apples and oranges.

Putting aside that this still doesn't identify the level change[2], any different weighting in that mean is still equally consistent with the data but yields different results. Consider if in period s < r, apples are preferred more, such that if the prices were equal, the consumer would buy more apples, and vice versa in period r. If the desires were normalized by dividing by a mean with more weight on oranges, both desires would be divided by a higher number, both would fall, the consumer would be worse off, and the COLI would rise. The reverse would happen if the mean had more weight on apples, even though the observed shares of apples vs. oranges would be the same in either case. Additional data on the desires for apples and oranges is needed.

A literature surveyed by Martin (2022) discusses changing preferences in a COLI, including Balk (1989), Caves, Christiansen, & Diewert (1982), Fisher & Shell (1972), Pollak (1975), and Feenstra & Reinsdorf (2007). This paper, however, is focused only on the issue of preference normalization.

Section 2 presents the issue formally using the RW framework as the primary example, and the identification problem with the normalization, and shows alternative normalizations. Section 3 presents the calculated indexes implied by the alternative normalizations and compares them. Section 4 concludes.

2 Indexes with Changing Preferences

RW models consumer preferences with constant elasticity of substitution utility, so the expenditure function is[3]

$$P_t^* = \left[\Sigma_{k \in \Omega_{t-1,t}} \left(\frac{p_{kt}}{b_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{1}$$

where $\Omega_{t-1,t}$ denotes the set of all goods common to both periods t-1 and t, p_{kt} is the price of good k in period t, σ is the elasticity of substitution between goods, and b_{kt} is a preference parameter for good k in period t, which could be interpreted as good quality, appeal, as a demand shifter, or specification error or just a demand residual. While Martin (2022)[4] shows that how the common

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is that the units of measurment do not matter for the inflation rate. However, all of the normalizations considered in this paper take this form, so that all preserve commensurability.

[2] See Martin (2023).

[3] This is RW equation (1) with notation of b_{kt} in place of ϕ_{kt}.

[4] This is shown in Appendix C of Martin (2022).

goods are defined in this framework changes the results, here I abstract from the choice of common goods.

The CES COLI is

$$COLI_{t-1,t} = \frac{P_t^*}{P_{t-1}^*} = \frac{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{b_k^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{k,t-1}}{b_k^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \tag{2}$$

for some base preferences b_k^*. The proposed RW index is based on[5],

$$\frac{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{b_{kt}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{k,t-1}}{b_{k,t-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \tag{3}$$

where the current period b_{kt} preferences are in the numerator and previous period preferences $b_{k,t-1}$ are in the denominator.

The Sato-Vartia (SV) index is an exact COLI for usual CES preferences[6], given by

$$\Phi_t^{*SV} = \Pi_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{p_{k,t-1}}\right)^{\omega_{kt}^*} \tag{4}$$

where ω_{kt}^* are the Sato-Vartia weights, $\omega_{kt}^* = \frac{\frac{s_{kt}^* - s_{k,t-1}^*}{\ln s_{kt}^* - \ln s_{k,t-1}^*}}{\Sigma_{k\in\Omega_{t-1,t}}\frac{s_{kt}^* - s_{k,t-1}^*}{\ln s_{kt}^* - \ln s_{k,t-1}^*}}$. The "taste-shock bias" is therefore defined as

$$\text{taste-shock bias} \equiv \ln\Phi_t^{*SV} - \frac{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{b_{kt}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{k,t-1}}{b_{k,t-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \tag{5}$$

.

However, the goods' shares in CES are[7]

$$s_{kt}^* = \frac{\left(\frac{p_{kt}}{b_{kt}}\right)^{1-\sigma}}{\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{b_{kt}}\right)^{1-\sigma}} \tag{6}$$

which implies[8]

$$(P_t^*)^{1-\sigma} = \frac{1}{s_{kt}^*}\left(\frac{p_{kt}}{b_{kt}}\right)^{1-\sigma} \tag{7}$$

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[5] This is equivalent to equation (9) in RW.

[6] If preferences change, the SV index is exact for preferences that are intermediate between the two periods. See Feenstra & Reinsdorf (2007).

[7] This is RW equation (2).

[8] This is equivalent to RW equation (3).

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Because all the b_{kt} parameters could be divided by a constant and yield the same shares and index, only the ratios $\frac{b_{kt}}{b_{jt}}$ are identified. If all b_{kt} parameters rose or fell, the index would have notional changes that cannot be identified. Therefore, for an index for periods t-1 to t, they are normalized to rule this out. Consider all normalized preference parameters ϕ_{kt} such that

$$\phi_{kt} \equiv \frac{b_{kt}}{\Pi_{k \in \Omega_{t-1,t}} b_{kt}^{w_{kt}}} \tag{8}$$

where w_{kt} denotes normalization weights such that $\Sigma_{k \in \Omega_{t-1,t}} w_{kt} = 1$. All such w_{kt} are equally consistent with the data in (6). Therefore, by definition,

$$\Pi_{k \in \Omega_{t-1,t}} \phi_{kt}^{w_{kt}} \equiv 1 \tag{9}$$

. To rule out notional index changes the b_{kt} are replaced with ϕ_{kt}.

Taking the log of both sides of (7), time differencing and rearranging yields

$$(1-\sigma) \ln \frac{P_t^*}{P_{t-1}^*} = (1-\sigma) \ln \frac{p_{kt}}{p_{k,t-1}} - (1-\sigma) \ln \frac{\phi_{kt}}{\phi_{k,t-1}} - \ln \frac{s_{kt}^*}{s_{k,t-1}^*} \tag{10}$$

. Multiplying by w_{kt} and summing over common goods, yields

$$(1-\sigma) \ln \frac{P_t^*}{P_{t-1}^*} = (1-\sigma) \Sigma_{k \in \Omega_{t-1,t}} w_{kt} \ln \frac{p_{kt}}{p_{k,t-1}} - (1-\sigma) \Sigma_{k \in \Omega_{t-1,t}} w_{kt} \ln \frac{\phi_{kt}}{\phi_{k,t-1}} - \Sigma_{k \in \Omega_{t-1,t}} w_{kt} \ln \frac{s_{kt}^*}{s_{k,t-1}^*} \tag{11}$$

and by (2)

$$\frac{P_t^*}{P_{t-1}^*} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}} \right)^{w_{kt}} \left[\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*} \right)^{\frac{w_{kt}}{\sigma-1}} \right] = \Pi_{k \in \Omega_{t-1,t}} \left(\frac{p_{kt}}{p_{k,t-1}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*} \right)^{\frac{1}{\sigma-1}} \right)^{w_{kt}} \tag{12a}$$

.

The RW CCV index, denoted Φ_t^{*CCV9}, uses an equally weighted geometric mean normalization, $w_{kt} = \frac{1}{N_{t-1,t}}$,[10]

$$\Phi_t^{*CCV} = \frac{\left[\Sigma_{k \in \Omega_{t-1,t}} \left(\frac{p_{kt}}{\phi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k \in \Omega_{t-1,t}} \left(\frac{p_{k,t-1}}{\phi_{k,t-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}} = \frac{P_t^*}{P_{t-1}^*} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}} \right)^{\frac{1}{N_{t-1,t}}} \left[\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*} \right)^{\frac{1}{N_{t-1,t}(\sigma-1)}} \right] \tag{13}$$

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[9] This is RW equation (9) without the tilde notation.
[10] The analogy in Zadrozny (2023) for the weights is the e^T vector normalizing the f_t terms.

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However, an unweighted mean puts as much importance on a very widely consumed good as one with a trivial share. Consider the normalization with the SV weights instead, $w_{kt} = \omega^*_{kt}$,

$$\Pi_{k \in \Omega_{t-1,t}} \phi_{kt}^{\omega^*_{kt}} \equiv 1 \tag{14}$$

. Then $(12a)$ becomes

$$\frac{P^*_t}{P^*_{t-1}} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}} \right)^{\omega^*_{kt}} = \Phi^{*SV}_t \tag{15}$$

because

$$\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{\omega^*_{kt}}{(\sigma-1)}} = 1 \tag{16}$$

. Therefore, taste-shock bias from (**??**) is exactly zero.

If the weights were last period's shares, $w_{kt} = s^*_{k,t-1}$, the index would be

$$\frac{P^*_t}{P^*_{t-1}} = G_{t-1,t} \left[\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{s^*_{k,t-1}}{\sigma-1}} \right] \tag{17}$$

where $G_{t-1,t} = \Pi_{k \in \Omega_{t-1,t}} \left(\frac{p_{kt}}{p_{k,t-1}} \right)^{s_{k,t-1}}$ is the geometric means index with base period (lag) shares.

Using current period shares, $w_{kt} = s^*_{k,t}$,

$$\frac{P^*_t}{P^*_{t-1}} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}} \right)^{s^*_{k,t}} \left[\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{s^*_{k,t}}{(\sigma-1)}} \right] \tag{18}$$

. Using Törnqvist weights, $w_{kt} = \frac{1}{2} \left(s^*_{kt} + s^*_{k,t-1} \right)$,

$$\frac{P^*_t}{P^*_{t-1}} = TQ_{t-1,t} \left[\Pi_{k \in \Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{\frac{1}{2}\left(s^*_{kt} + s^*_{k,t-1} \right)}{\sigma-1}} \right] \tag{19}$$

where $TQ_{t-1,t}$ denotes the Törnqvist price index. In each case, (**??**) differs.[11]

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[11]Zadrozny (2023) uses generalized CES preferences instead, which allow for non-homotheticity. However, the same normalization (**??**) is required, where the α_{it} parameters are analogous to the ϕ_{kt} RW parameters.

3   Comparison of Calculated Indexes with Alternate Preference Normalizations

CCV type indexes with alternative normalizations were constructed to compare to the CCV and SV indexes. The data used was chosen to be as similar as possible to RW. However, RW use AC Nielsen's Homescan data, which is proprietary data of a household survey of purchases from grocery and drug stores in the U.S. Homescan data for the same time span as RW, 2004-2014, was not available for this study. Instead, I use BLS's five years of data of AC Nielsen's Scantrak data, which contains the universe of transactions data of all large grocery and drug store chains in the U.S., so this is not meant to replicate the exact results. The data contains weekly and monthly total quantities and unit value prices for goods defined at the detail of UPC, which generally identifies an exact good. Following RW, the data is aggregated to the quarterly level, and index changes from 4th quarter to 4th quarter were calculated, from 4th quarter 2005 - 4th quarter 2009. The data is divided into categories called product groups, which were aggregated using a Laspeyres formula with 4th quarter 2005 as the base period. The elasticities of substitution were estimated separately for each product group, using the methodology in Broda & Weinstein (2010), which in turn uses the Feenstra (1994) method, to be as consistent as possible with RW.

Figure 1[12] and the table below reports the index changes for the CCV, Fisher, Sato-Vartia share normalization (SV index), Fisher, Törnqvist share normalization, lagged share normalization, and current (lead) share normalization. The "taste change bias" is the difference between these indexes and the SV index. As shown above, the normalization using SV weights is exactly the SV index, with zero bias. The standard Fisher index is very close, and the Törnqvist share index is slightly below. As reported in RW, the equal shares index and lag shares index are greatly below the SV index, and except for the first year, similar to each other, implying upward bias in a SV index. However, the lead shares index is the opposite, being greatly higher and implying downward bias in the SV index. These differences are very significant since they compound over time. For example, the average difference between the lead share index and the SV is 1.5%/year, which over ten years would compound to a difference in index levels of 16.25%.

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[12] These results are analogous to RW Figure V, and the lagged share normalization is what they refer to as the "CUPI - Initial Shares". RW include the Fisher index for comparison.

Figure 1: Index Relatives with Different Preference Normalizations, 4th qtr to 4th qtr
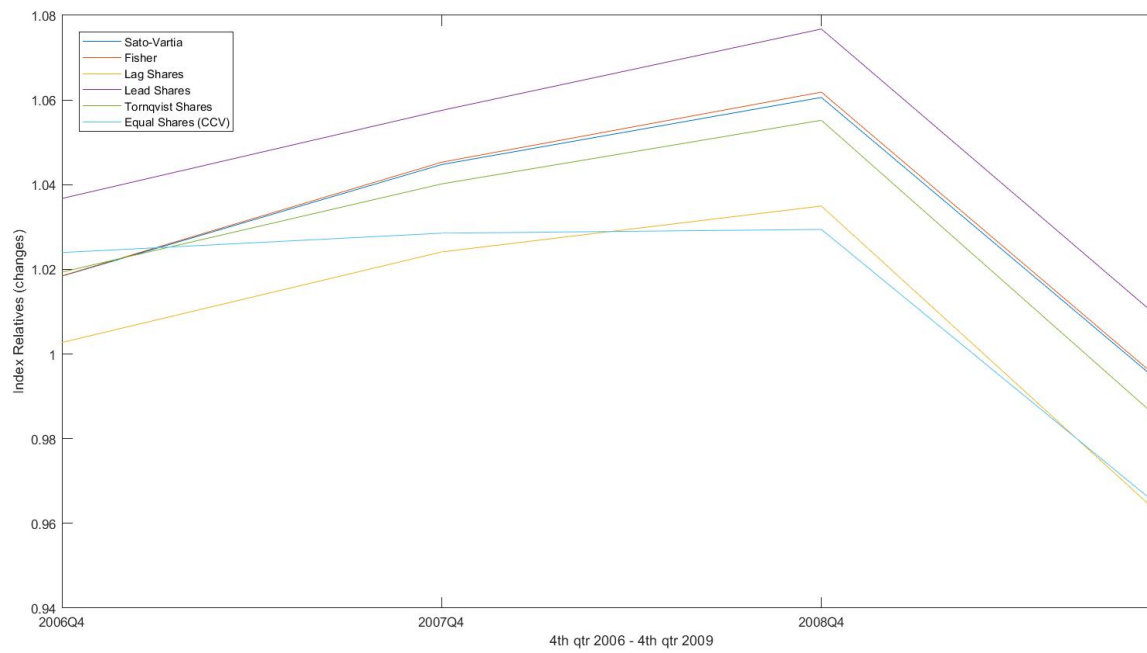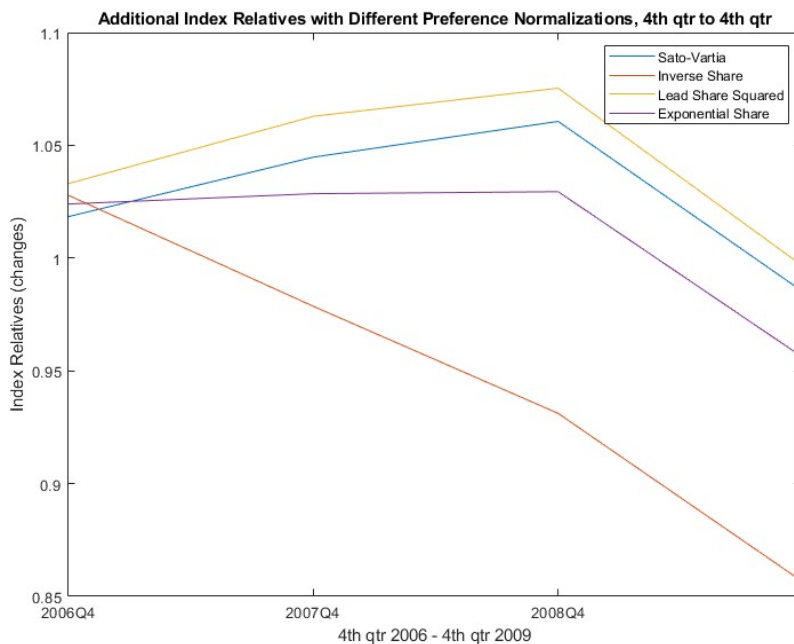

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Figure 2: Additional Index Relatives with Different Preference Normalizations, 4th qtr to 4th qtr

Additional Index Relatives with Different Preference Normalizations, 4th qtr to 4th qtr

Index	Period			
	2006Q4	2007Q4	2008Q4	2009Q4
Sato-Vartia	1.0183	1.0447	1.06060	0.9851
Fisher	1.0184	1.0453	1.06186	0.9857
Lag Shares	1.0027	1.0241	1.03495	0.9532
Lead Shares	1.0367	1.0575	1.07676	1.0000
Törnqvist Shares	1.0193	1.0401	1.05521	0.9758
Equal Shares (CCV)	1.0239	1.0285	1.02943	0.9559
Inverse Share	1.0278	0.9786	0.93114	0.8564
Lead Share Squared	1.0330	1.0628	1.07532	0.9964
Exponential Share	1.0239	1.0285	1.02945	0.9559

Additional normalizations are shown in Figure 2 and the table, including weighting by the inverse share, squared lead share, or and exponential of the share, for weights of $w_{kt} = \frac{\frac{1}{\omega_{kt}^*}}{\Sigma_k \frac{1}{\omega_{kt}^*}}$, $\frac{s_{kt}^{*2}}{\Sigma_k s_{kt}^{*2}}$, and $\frac{\exp(1+s_{kt}^*)}{\Sigma_k \exp\left(1+s_{kt}^*\right)}$ respectively. The inverse and exponential share weighted indexes imply upward bias in the SV of greatly different magnitude, while the lead share squared index implies the opposite.

More generally, a weighted mean always lies between the minimum and maximum values, depending on the weights. The index is a weighted geometric mean of

$$x_{kt} = \frac{p_{kt}}{p_{k,t-1}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{1}{\sigma-1}} \tag{20}$$

. Therefore,

$$x^{\min}_{kt} \leq \frac{P^*_t}{P^*_{t-1}} \leq x^{\max}_{kt}$$

. The highest x_{kt} over good and time, $x^{\max}_{kt}$, is about 4800%, while the lowest x_{kt}, $x^{\min}_{kt}$, is about 86% deflation. Therefore, to yield the desired index I^{choice}_t, pick a weighting on the maximum x_{kt}, $w^{\max}_{kt}$ such that

$$I^{choice}_t = \left(x^{\min}_{kt} \right)^{w^{\max}_{tk}} \left(x^{\min}_{kt} \right)^{1-w^{\max}_{tk}} \tag{21}$$

and place a trivial weight on all other x_{kt} values,

$$w_{kt} = \begin{array}{ccc} w^{\max}_{kt} & if & k \text{ is max} \\ 1 - w^{\max}_{kt} & if & k \text{ is min} \\ \varepsilon & otherwise \end{array} \tag{22}$$

. Thus, if a statistical agency wanted to incorporate changing preferences by this method, they could always claim they were doing it by choosing a set of weights that yielded exactly the same indexes that they produced before!

A simple example shows how different weights affect the index. "Taste-Shock Bias" is only positive for the equal weighting normalization if product sales shares diverge, as can be seen from (13) when the geometric mean of the shares rises between periods. The problem with the equal weighting normalization is that smaller shares have a proportionately higher relative change. Suppose there are only two goods, with shares $= .8$ and $.2$, which then change to $.9$ and $.1$, with $\sigma = 4$. The second term in the r.h.s. of $(12a)$ becomes

$$\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{w_{kt}}{\sigma-1}} = \left(\frac{.9}{.8} \right)^{\frac{.5}{3}} \left(\frac{.1}{.2} \right)^{\frac{.5}{3}} = (1.019\,8)(0.890\,90) = 0.908\,56 < 1 \tag{23}$$

. Higher weighting on higher share goods could reverse this, such as $w_{kt} = s^*_{kt}$, so that

$$\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{w_{kt}}{\sigma-1}} = \left(\frac{.9}{.8} \right)^{\frac{.9}{3}} \left(\frac{.1}{.2} \right)^{\frac{.1}{3}} = (1.036\,0)(.977\,16) = 1.012\,3 > 1 \tag{24}$$

. The weight on the higher share goods therefore determines whether this term moves the index upwards or downwards. Depending on how share ratios are correlated with prices ratios (determined by the elasticity), this determines how the index compares to a SV index.

4 Conclusions/Discussion

Since any index could be rationalized by the choice of a particular set of weights, the decision to adjust for changing preferences by normalizing them is empty. In order to account for changing preferences requires data on how and why preferences change. All official COLIs that are produced have unobserved parameters or prices, meaning that they are conditional COLIs. In order to compare costs of living for a given standard of living over two periods, implicitly there is the same preference function with changing parameters or unobserved price: $e\left(p_t, \bar{u}; \phi_t\right)$ vs. $e\left(p_\tau, \bar{u}; \phi_\tau\right)$ with changing ϕ_t in the RW framework. Additional data on the ϕ_t terms is needed to identify them. For example, the change in the ϕ_t term for apples might be identified with data on previously unobserved qualities of apples. But prices and quantities of goods are not sufficient.

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