Price Dispersion and Cost-Of-Living Indexes

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Abstract

Recently, several Konüs cost-of-living indexes that allow stochastic prices have been described. However, under some conditions these indexes may violate an identity axiom and allow utility to increase with the measured cost of living. As an alternative I use literature on consumer surplus to describe a new Konüs index that does not violate the identity axiom and ranks price regimes in the opposite order as indirect utility functions. In addition, it provides a natural way to introduce risk aversion into cost-of-living indexes. This is demonstrated by examining the implication of risk aversion on the cost-of-living from fixed sample size searches.

I. Introduction

Standard cost-of-living indexes assume that prices in each of two time periods are nonstochastic. Research, however, suggests that prices are in fact dispersed across outlets, even after accounting for quality differences.[1] In a world of zero search costs this dispersion is irrelevant because every household will locate the minimum available price. More realistically, when search costs are positive, each household faces a distribution of offered prices, and the prices actually paid will vary across households. Therefore, an index based on nonstochastic prices has few applications in the real world and isn't necessarily related to indexes calculated with stochastic prices.

Baye (1985), however, describes a Konüs index for random prices by taking expectations over the indexes of individual agents. He also shows that his index obeys several appealing axioms. Reinsdorf (1994a) notes a limitation with Baye's index and suggests an alternative in which expectations are taken separately over the expenditure functions in each time period. He then uses his index to examine the cost-of-living when households use a fixed sample size search rule.

This paper draws upon research on consumer surplus to describe two additional complications with these indexes. These new complications have not been previously described because the level of utility is left undetermined. When the utility level is set by an indirect utility function the above indexes can collapse into essentially the same index that can violate an identity

[1] See, for example, Van Hoomissen (1988).

axiom. Failure to obey this axiom implies that such an index can indicate an increase in the cost of living when the distribution of prices is actually stationary. In addition, utility may increase when the cost of living index increases. Given this possibility, the index is arguably of little value because households may prefer a higher level of the index to a lower one.

As an alternative, I use the expenditure functional concept of Helms (1985), developed for a measure of expected compensating variation, to create a new Konüs index. This index describes the amount of money in a later time period necessary to equate expected indirect utility in both periods, which offers several advantages. First, it does not violate the above-mentioned identity axiom. Second, it ranks price regimes in the opposite order as expected indirect utility functions. Therefore, it is a useful measure of welfare changes. Finally, because it uses expected indirect utility, it provides a natural way to introduce risk aversion into cost-of-living indexes. For example, the gains or loses from increasing the spread of prices will depend upon household risk-loving behavior in prices.

To show some of the implications of introducing risk aversion into a cost-of-living index, the fixed sample size search cases in Reinsdorf (1994a) are considered. The two major results are: First, that for the index introduced here, risk aversion, not just search, determines whether or not a consumer benefits from increased price dispersion. Second, Reinsdorf's intuition that a high elasticity of substitution decreases the relative gains from search is given a precise meaning.

II. The Standard Konüs Index

A cost-of-living index is a useful way to consider welfare changes caused by changees in factors exogenous to the individual household, such as the monetary and trade policies of governments. By describing the effect of changes in terms of the percent of income necessary to leave the household indifferent, it provides a unit-free measure of the change in welfare. Price indexes, such as the U.S. Consumer Price Index, are approximations of these indexes.

To describe a cost-of-living index, first we need to assume that an individual household has a well-behaved utility function and a budget constraint. These generate a Marshallian demand function x(p,y), a Hicksian (compensated) demand function h(p,u) and an expenditure function e(p,u), where p is an m-dimensional vector of nonstochastic prices, $m > 1$, y is a fixed level of income, and u is some level of utility. The standard definition of a Konüs cost-of-living index is the ratio:

$$K(p,q,u) = e(p,u)/e(q,u) \qquad (1)$$

where q represents prices in the earlier reference period and p represents prices in the later comparison period. The index K shows the percentage change in income in the comparison period necessary to return the individual to the same level of utility as in the reference period. This use of utility information is the crucial feature distinguishing cost-of-living indexes from ‘mechanical’ price indexes.

As a consequence of this distinction, any specific price index formula is an exact cost-of-living index only for a proper subset of utility functions. This implies that no price index formula is ideal for all utility functions. Desirable axioms for price indexes to follow have been created, however, and they can help researchers sort meaningfully among price indexes. Baye (1985) suggests some analogous axioms for cost-of-living indexes when prices are random. The nonstochastic versions for a cost-of-living index I(p,q,u) are:

Monotonicity

For an m-dimensional vector q^{λ} with element j equal to q_j for all $j \neq i$ and element i equal to $q_i\lambda$, with $\lambda>1$, $I(q^{\lambda},q,u)>1$ and $I(q, q^{\lambda},u)<1$.

Homogeneity

For $\lambda>0$, $\lambda I(p,q,u)=I(\lambda p,q,u)$

Dimensionality

For $\lambda>0$, $I(\lambda p,\lambda q,u)=I(p,q,u)$

Identity

$I(q,q,u) = 1$.

The standard Konüs satisfies all of these properties. The first axiom states that, if all prices but one stay constant, then if that price increases/decreases, the index must be greater/less than unity. This property forces indexes to show an increase in the cost of living if any price increases. Without it, prices could rise, while the index indicates a decrease in the cost of living. (Linear) homogeneity just states that increasing all prices in the second period by the same proportion should increase the total index by that proportion. Dimensionality forces indexes with proportional prices to equal each other. The identity axiom forces indexes with identical prices to show no change in the cost of living. Without this feature, the cost of living can appear to rise or fall even if prices and utility do not changes.

Note that, while utility u is explicitly used, the source of utility is not specified in either the index or in the axioms. Nevertheless, because the expenditure function e(p,u) is the inverse of the indirect utility function v(p,y), the Konüs also satisfies an additional useful axiom:

Preference Monotonicity

sgn(I(p,q,u) - 1) = sgn(v(p,y) - v(q,y)).

Without this axiom, it is difficult to make normative statements about the cost of living. For example, one cannot conclude that a higher cost of living is undesirable because an individual may prefer it.

III. Konüs Indexes Under Price Dispersion

Since the assumptions usually made to create a Konüs index (e.g., nonstochastic prices) are highly restrictive, it is not surprising that the standard Konüs satisfies the above axioms. One possible weakening of these assumptions is to allow for the random dispersion of prices. This could represent an array of retail outlets offering different prices to a set of otherwise identical consumers with limited information who search over prices at some cost or who search a fixed number of times. At least two distributions are of interest here. First, the distribution of prices that the consumer could possibly encounter i.e., the offered prices. This is the relevant distribution when the Bureau of Labor Statistics collects price quotes from outlets to form the consumer price index. A second distribution of interest would be a distribution over prices located by consumers, i.e., the distribution of accepted, or transacted, prices.

Any index constructed from prices with these distributions will have to aggregate across both prices and goods. This implies that the axioms above could not be applied because they are designed solely in terms of goods aggregation. But without any underlying theory as guidance, the construction of an index over stochastic prices is necessarily ad hoc. In addition, intuition developed for indexes such as that in equation (1) may be useless.

As a step to developing a cost-of-living index methodology when prices are stochastic, Baye (1985) proposes an index that can be used with the distribution of transacted prices. Rather than simply use the expectation of prices in place of fixed prices, Baye takes expectations over the whole index. This Expected Konüs (EK) index is defined by

(2) $$EK(\Omega_{p,q},u) = E\left[e(\tilde{p},u) \big/ e(\tilde{q},u) \right].$$

where the expectation is taken over the random price vectors $\tilde{p}$ and $\tilde{q}$ associated with the probability space $\Omega_{p,q}$. One interpretation of EK is that it minimizes the expected squared deviation from a randomly selected consumer's Konüs index of transacted prices. It also equals the expected index of offered prices under the assumption that the consumer does not change outlets in the second period. Baye states that EK obeys useful axioms adapted for cost-of-living indexes with random prices.

These axioms, and the random price version of preference monotonicity are:

<u>Monotonicity (random)</u> Define Ω_{p,q^λ} as the probability space caused by the random price vectors $\tilde{p}$ and $\tilde{q}^{\lambda}$, where element i of $\tilde{q}^{\lambda}$ equals $\lambda \cdot q_i$, with λ being a constant greater than unity. Then for some index EI of random prices, $EI(\Omega_{q^\lambda,q},u) \geq EI(\Omega_{q,q},u)$ and $EI(\Omega_{q,q},u) \geq EI(\Omega_{q,q^\lambda},u)$.

<u>Homogeneity (random)</u>

Define $\Omega_{\lambda p,q}$ as the probability space of $\tilde{q}$, $\lambda \cdot \tilde{p}$. For $\lambda>0$ $\lambda EI(\Omega_{p,q},u)=EI(\Omega_{\lambda p,q},u)$.

<u>Dimensionality (random)</u>

For $\lambda>0$, $EI(\Omega_{\lambda p, \lambda q},u) = EI(\Omega_{p,q},u)$

<u>Identity (random)</u>

$EI(\Omega_{q,q},u)=1$.

<u>Preference Monotonicity (random)</u>

$sgn(EI(\Omega_{p,q},u) - 1) = sgn(E[v(p,y)] - E[v(q,y)])$.

For a given household, let the Konüs index be $K(\lambda_i q,q,u)$, where λ_i is the realization of the random variable $\tilde{\lambda}$ for household i. Baye shows that, if $E[\tilde{\lambda}]=1$ so that $E[\tilde{p}]$ equals q, then EK is less than one. This follows from the concavity of the expenditure function with respect to prices and from Jensen's inequality.

As a result, the cost of living always falls with the introduction of price dispersion, which raises the issues of risk aversion generated by Waugh (1944), although it has received little attention in the literature on cost-of-living indexes. In the present context it seems counterintuitive that the cost of living always falls with the introduction of price dispersion because this seems to say that households would always prefer greater dispersion to less dispersion. On the contrary, one might expect that there is a level of risk aversion that makes an increase in dispersion undesirable.

Reinsdorf (1994a) touches indirectly on this issue by pointing out that although EK satisfies the identity axiom above for accepted prices, it may not satisfy the identity axiom for the distribution of offered prices if the household searches in both periods. He gives the following example:

> [S]uppose that a single good is consumed and that offered prices in both period 0 and period 1 are distributed such that after searching consumers have equal probabilities of paying $1 or $2. Then EK will equal 0.5·(0.25)+1·(0.5) +2·(0.25), or 1.125. In order for EK to satisfy the identity property, it must be the case that whenever offered prices have the same distribution all consumers locate exactly the same price that they did in the reference period.

As a solution to this problem Reinsdorf describes an index which he titles the ratio of expectations index and will be referred to as the REK index here, for the case where prices are random variables due — for example — to a consumer facing dispersed prices in retail markets.

This index describes the ratio of expected expenditures over offered prices in both periods. The REK index is defined as:

(3) $$REK(\Omega_{p,q}, u) = \frac{E[e(\tilde{p}, u)]}{E[e(\tilde{q}, u)]}$$

where the expectation is taken over the prices in the first argument of the expenditure function.

IV. New Complications

While the REK index is designed to measure the changes in offered prices, there are potential hazards when using an expectation operator on expenditure functions. For example, under price dispersion, the expenditure minimization problem implied by the REK has an unsettling dual: a utility maximization problem in which prices and income are random variables, with income varying in such a way as to hold utility constant.

This difficulty exists because the source of the base utility level u isn't specified. If instead one assumes that this level is the outcome of a standard utility maximization program, then utility will itself vary with different prices. Income can then be thought of as fixed, which implies that expenditures in the base period are also fixed. Now u can be replaced with the indirect utility function v(b,y), where b represents prices in some base period. Substituting the indirect utility function into the expenditure function, the Konüs cost-of-living index becomes

$$K^*(p,q,b,y) = e(p,v(b,y))/e(q,v(b,y))$$

$$= a(p,b,y)/a(q,b,y).$$

The index K*(p,q,y) can be interpreted as the percent change between p and q in the cost of maintaining the level of utility attained with base prices b and income y. The function a(p,b,y) is the amount of income necessary to maintain the same level of utility with prices p, as is possible with prices b and income y. Appendix A describes several useful relationships involving a(p,b,y) and v(b,y).

Although in principle base prices can be any period, a logical selection is the reference period associated with prices q. Diewert (1993) defines a Konüs index using this base period as a Laspeyres-Konüs index, and it will be the focus of this paper. Setting b equal to q immediately yields

(4) K*(p,q,y)=e(p,v(q,y))/e(q,v(q,y))

=a(p,q,y)/y.

This shows that the calculation of the Konüs index involves only a single expenditure and utility function. The second equality in (4) holds by P1 and P5 of Appendix A. Because q and y are the same in both the numerator and denominator, utility is the same in both time periods. The index K*(p,q,y) is then equal to the percent of expenditures with prices p necessary to achieve the utility achieved with prices q and income y.

To form the random price analog of K*, transform EK by the substitution of v(q,y) with u.[2] The expected Konüs index is then:

(5) $$EK^*(\Omega_{p,q}, y) = E\left[a(\tilde{p}, \tilde{q}, y) \big/ a(\tilde{q}, \tilde{q}, y) \right] = E\left[a(\tilde{p}, \tilde{q}, y) \big/ y \right]$$

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[2] As with K*, there is no requirement that the base period for utility be the same as the period with prices q. The generalization of REK* and EK* to an arbitrary base period are

$$EK^*(\Omega_{p,q,b}, y) = E\left[\frac{\mu(\tilde{p}, \tilde{b}, y)}{\mu(\tilde{q}, \tilde{b}, y)} \right]$$

and

$$REK^*(\Omega_{p,q,b}, y) = \frac{E[\mu(\tilde{p}, \tilde{b}, y)]}{E[\mu(\tilde{q}, \tilde{b}, y)]}.$$

and the ratio of expectations index is:

(6) $$REK^*(\Omega_{p,q},y) = \frac{E[a(\tilde{p},\tilde{q},y)]}{E[a(\tilde{q},\tilde{q},y)]} = \frac{E[a(\tilde{p},\tilde{q},y)]}{y}$$

Because y is a constant and base period prices are $\tilde{q}$, the only distinction between REK* and EK* lies in interpreting the probability measure as applying to offered or transacted prices. Given the probability measure, the calculation of the two statistics is otherwise identical. One can then consider many characteristics of the expected expenditure income ratio $EIR(\Omega_{p,q},y) = E[a(\tilde{p}, \tilde{q}, y)]/y$, without having to specify whether the index is REK* or EK*.

For example, one can consider how EIR satisfies the cost-of-living axioms:

Monotonicity (random)*

Define $\Omega_{p,q}\lambda$ as before. Then for some index $EI^*(\Omega_{p,q},y)$, $EI^*(\Omega_{q}\lambda_{,q},y) \geq EI^*(\Omega_{q,q},y)$ and $EI^*(\Omega_{q,q},y) \geq EI^*(\Omega_{q,q}\lambda,y)$.

Homogeneity (random)*

Define $\Omega_{\lambda p,q}$ as before. For $\lambda>0$, $\lambda EI^*(\Omega_{p,q},y)=EI^*(\Omega_{\lambda p,q},y)$.

Dimensionality (random)*

For $\lambda>0$, $EI^*(\Omega_{\lambda p,\lambda q},y) = EI^*(\Omega_{p,q},y)$

Identity (random)*

$EI^*(\Omega_{q,q},y)=1$.

Preference Monotonicity (random)*

$sgn(EI^*(\Omega_{p,q},y) - 1) = sgn(E[v(p,y)] - E[v(q,y)])$.

The following theorem shows that the identity axiom fails to hold for EIR, regardless of the probability measure of the index:

Proposition 1) If the prices for a normal good are random draws from a nondegenerate IID distribution within both periods then EIR($\Omega_{p,q}$,y) >1.

Proof: See Appendix B

By specifying REK* and EK* as EIR($\Omega_{p,q}$,y), we can also use a theorem in Helms (1985) to examine preference monotonicity. Because this theorem describes the conditions under which E[e(p̃,v(q,y)]-y has the same sign as v(q,y)- E[v(p̃,y)], it also gives the conditions under which EIR - 1 has the same sign as v(q,y)- E[v(p̃,y)]. If these conditions fail to hold, then EIR > 1 may accompany an increase in expected utility and households actually prefer the cost of living, as measured by EIR, to rise.

Theorem 1 (Helms 1985): Assume that v(p,y) is an indirect utility function of two goods, one with price q in the base and random price p̃ in the second period and the second a composite numeraire good, and income y. If

$\text{sgn}(E[e(\tilde{p},v(q_1,y))]-y)= \text{sgn}(v(q_1,y)-E(v(\tilde{p},y))$

then for any p and y with x(p,y)>0

i) $v_{yp}=x_y v_y$

ii) $\rho^y=2\eta^y$

iii) $\rho^p=-\varepsilon^h$,

where

$$\rho^y = \frac{-v_{yy}}{v_y} y$$ *, the relative level of income risk aversion*

$$\rho^p = \frac{-v_{pp}}{v_p} p$$ *, the relative level of price risk aversion*

$$\eta^y = x_y(p,y)\frac{y}{x(p,y)}$$, *the income elasticity of demand*

$$\varepsilon^h = h_p(p,u)\frac{p}{h(p,u)}$$, *the compensated price elasticity of demand*

with the subscripts on functions denoting partial derivatives.

For expected expenditures to always decrease while expected utility increases, households must be risk averse in income in a very specific manner, while being risk loving in prices in another specific manner. For all other preferences sets, there will exist $\tilde{p}$ and q such that the change in expected utility will move in the same direction as the change in the EIR index. By continuity, one can also apply this argument to random q as well.

The intuition for the result is straightforward: while the expenditure function makes use of ordinal information about utility, risk aversion uses cardinal properties. This means that agreement between the expenditure function and the indirect utility function imposes restrictions on the allowable types of risk aversion.

Such lack of welfare properties makes the cost-of-living index of little value in this case. An EIR index is still generally useful for examining the cost of living increase moving from a fixed price regime to a variable price regime where the mean prices are the same as the fixed prices in the initial regime. It can be shown that in this case the condition necessary for a decrease in the EIR index to always accompany an increase in expected indirect utility is that ρ^p be greater than zero. Turnovsky, Shalit and Schmitz (1980) show that this is not unusually restrictive because a sufficient condition for ρ^p to be positive is that $2\eta^y - \rho^y \geq 0$. For example, this holds in constant elasticity of substitution utility functions because η^y is equal to one while ρ^y equals zero.

V. A New Konüs Index Under Price Dispersion

The previous section describes problems when explicitly setting base period utility in the expenditure function when prices are random. This section suggests a possible solution that also allows the incorporation of risk aversion into cost-of-living indexes. Once done, the large body of literature on expected consumer surplus measures can then be applied to consumer price indexes. As an example of the implications of introducing risk aversion, let us examine two cases described in Reinsdorf (1994a).

Before addressing risk aversion an additional idea is needed. This concept, as described in Helms (1985), is the ex ante expenditure functional. Denoted as $\hat{e}(\Omega_q,v)$, it is the solution to the following problem:

Minimize y

subject to $E[v(\tilde{q},y)] \geq u$.

This functional is the amount of money required to achieve a given expected level of utility and can be calculated by inverting $E[v(\tilde{p},y)]$. It is 'ex ante' in the sense that the random price is realized before the expenditure function is formed. For a cost of living index one might still wish to know how much money is necessary to restore utility to this level if the price distribution changes. That is, it is of interest to ask what level of income $\ddot{y}$ is necessary to satisfy $E[v(\tilde{p},\ddot{y})] = E[v(\tilde{q},y)]$. Solving for $\ddot{y}$ yields $\hat{a}(\Omega_{p,q}, y)$. The implied ex ante Konüs index is

$$(7) \qquad EAK(\Omega_{p,q}, y) = \frac{\hat{a}(\Omega_{p,q}, y)}{y} .$$

To show that EAK satisfies the modified Eichhorn-Voeller axioms of monotonicity, homogeneity, dimensionality and identity, note that by considering the price of the same good in different states as separate prices on separate goods, $E[v(\tilde{q},y)]$ can be considered to be an indirect utility function itself. Therefore, EAK satisfies these axioms for the same reason that the standard Konüs satisfies them.

While clearly in the spirit of the REK index, EAK offers several advantages. First, the maximization problem generating $\hat{e}(\Omega_p,v)$ and $\hat{a}(\Omega_{p,q}, y)$ is sensible. Second, the identity property is satisfied because we are only taking expectations with respect to the indirect utility function. Third, the index follows the preference monotonicity axiom. Helms (1985) shows this for the compensating variation case:

Theorem 2 (Helms 1985): Given two random price vectors $\tilde{p}$ and $\tilde{q}$ fixed utility u and income y

$$\mathrm{sgn}(\hat{e}(\Omega_p,v) - \hat{e}(\Omega_q,v))=\mathrm{sgn}(E[v(\tilde{q},y)]- E[v(\tilde{p},y)]).$$

From this theorem it therefore holds that

$$\mathrm{sgn}(EAK(\Omega_{p,q},y)-1)=\mathrm{sgn}(E[v(\tilde{q},y)]- E[v(\tilde{p},y)]).$$

The final advantage of this index is that it incorporates risk aversion. The change in the index depend upon how the household views risk because the amount of income necessary to raise the expected indirect utility level will depend upon the level of risk aversion imbedded in the indirect utility function.

As an example, consider the cases of increased in price dispersion in Reinsdorf (1994a). Following Reinsdorf, assume that the cumulative distribution function $\Phi(p;z)$ of offered prices is twice continuously differentiable. Also, assume that for all z the function $\Phi(p;z)$ has a positive support (B,T), $B < \infty$ and for at least one point in the support $\Phi_z \neq 0$. Define

$$Q(r;z) \equiv \int_B^r \Phi_z \, dp$$

Then the Rothschild-Stiglitz conditions for an increase in z to cause a mean-and-support-preserving (MASP) increase in the dispersion of p are that (i) $Q(r;z) \geq 0$ for $r \in (B,T)$ and (ii) $Q(B;z) = 0$.

Assume that the consumer engages in a fixed sample size (FSS) search where the search over prices p_i occurs n_i times. Nonsearchers for this good are defined as those for whom n_i equals one. Reinsdorf states that the cumulative distribution function for the minimum prices after n_i searches of prices with cumulative distribution function $\Phi(p;z)$ is $1-[1-\Phi(p;z)]^{n_i}$. The probability density function is then $n_i\phi(p;z)[1-\Phi(p;z)]^{n_i -1}$, where $\phi(p;z)$ is the probability density function of p. Reinsdorf shows that, for a single good, FSS search is a sufficient condition for a MASP increase in price dispersion to imply REK <1. The equivalent proposition for multiple goods and EAK is:

Proposition 2) Assume the distribution of the price of good one is independent of prices $p_2,\ldots,p_m \equiv p\_$, that good one has positive demand and that all prices p_i have positive support $[B_i,T_i]$. *Then a sufficient condition for a MASP increase in the dispersion of p_1 to cause $EAK(\Omega_{p,q},y) <1$ for searchers and non-searchers is* $\rho^{p_1} \geq 0$. *FSS search is itself neither necessary nor sufficient. If* $\rho^{p_1} < 0$, *then there exists some distribution function and MASP increase in price dispersion such that $EAK(\Omega_{p,q},y) >1$ for searchers.*

Proof: See Appendix B

In this case FSS search does not necessarily cause $EAK(\Omega_{p,q},y) < 1$. The more important condition is that $\rho^{p_1}>0$, i.e., that the searcher is risk-loving in prices. While $\rho^{p_1}>0$ is sufficient, it is not necessary. A weaker condition for an increase in dispersion to cause $EAK(\Omega_{p,q},y) < 1$ is that $-E(v_1|p_1)$ is everywhere greater than $1/(1-\Phi(p;z))^{n_1 -1}$, where v_1 is the partial derivative of the indirect utility function with respect to the price of good one. This limit allows searchers to be risk-averse in the price p_1 but only up to some point determined by the specific distribution function. Hanoch (1977) shows that, if all income elasticities are equal to unity and $\rho^y>2$, then expected utility increases from a MASP increase in all prices.

Reinsdorf also shows that in the multi-good case a MASP increase in dispersion may reduce the REK index for nonsearchers more than searchers, even for search samples as small as two. He points out that this occurs because high levels of commodity substitution reduce the distinction between a consumer visiting markets for multiple goods and visiting multiple markets for the same good. Comparisons of nonsearchers with FSS searchers is then similar to comparisons between FSS searchers with different numbers of searches. In these comparisons the searchers with large n_i do not necessarily reduce, on the margin, expected expenditures more than nonsearchers when price dispersion increases. While the same intuition holds for EAK($\Omega_{p,q}$,y), a more precise connection between substitutability and the gains from price dispersion can be made.

Proposition 3) Consider the case in proposition 2, and let s_i *be the expenditure share for good i. If either* $|\varepsilon^h| > s_1(2\eta^y - \rho^y)$ or $2\eta^y - \rho^y > 0$ *then a MASP increase in dispersion benefits nonsearchers more than searchers. If searchers benefit more than nonsearchers, then either* $|\varepsilon^h| \leq s_1(2\eta^y-\rho^y)$ or $2\eta^y-\rho^y \leq 0$ *over some price range*.

Proof: See Appendix B.

This proposition explicitly lays out the connection among search, elasticities and risk aversion. For example the first sufficient condition for dispersion to relatively benefit nonsearchers implies that the greater the level of risk aversion in income the lower the compensated price elasticity of substitution necessary for nonsearchers to relatively benefit. This occurs because the marginal benefits of search are reduced with greater risk aversion. Greater risk aversion must then be accompanied by greater elasticity or searchers will be the relative winners. The second sufficient condition states that, beyond a sufficiently high level of risk aversion ($\rho^y=2\eta^y$), nonsearchers are relative winners regardless of the compensated elasticity of substitution. Also, note that income elasticity works in the opposite direction as compensated price elasticity. Finally, if preferences are Leontief, then a MASP increase in price dispersion benefits nonsearchers only if $2\eta^y>\rho^y$.

VI. Neoclassical Utility: A Useful Special Case

Diewert (1993) defines a useful set of utility functions whose elements U(x) are neoclassical if U(x) is (i) continuous, (ii) positive, i.e., U(x) > 0 if each element of x is positive, and (iii) linearly homogeneous, i.e., $U(\lambda x) = \lambda U(x)$ if $\lambda>0$. This class of utility functions is useful in clarifying the relation between the EAK index and the REK index by using the fact that the indirect utility function has the form $v(p,y) = y\cdot v(p,1)$. In this case, the expenditure function e(p,u) is equal to u/v(p,1), $a(\tilde{p},\tilde{q},y) = y\cdot v(\tilde{p},1)/v(\tilde{q},1)$ and $\hat{a}(\Omega_p, \Omega_q, y) = y\cdot E[v(\tilde{p},1)]/E[v(\tilde{q},1)]$. The indexes EAK and REK* can then be expressed as

$$\text{(8)} \qquad EAK(\Omega_{p,q}, y) = \frac{\dfrac{y\cdot E[v(\tilde{q},1)]}{E[v(\tilde{p},1)]}}{y} = \frac{E[v(\tilde{q},1)]}{E[v(\tilde{p},1)]}$$

and

$$\text{(9)} \qquad REK^*(\Omega_{p,q}, y) = \frac{y\cdot E\left[v(\tilde{q},1)\big/v(\tilde{p},1)\right]}{y} = E\left[\frac{v(\tilde{q},1)}{v(\tilde{p},1)}\right].$$

It should be clear that the EAK index in (8) is inversely related to the indirect utility function so that the EAK index only indicates an increase in the cost of living if the expected indirect utility function decreases. Then EAK-1 varies inversely with $E[v(\tilde{p},1)] - E[v(\tilde{q},1)]$ and Theorem 2 holds.

Theorem 1, however, still constrains an equivalent relationship for REK*. This applies if the expectation operator generates the arithmetic mean of the random variable. In many cases, however, we are interested in generating the geometric mean instead. For example, Flemming et. al. (1977) show that the geometric mean avoids potential problems with the selection of a numeraire good.

Proposition 4) Assume that utility is neoclassica and define the geometric mean of x as exp(E[ln(x)]). Then if expectations are taken using a geometric mean, EAK=REK.*

To see this note that $E[1/\chi]=1/E[\chi]$ for the geometric mean operator. In general, one cannot sign the difference between REK* and EAK. For the neoclassical case, however, it is positive by Jensen's inequality.

While both risk aversion and risk loving in prices is possible with neoclassical preferences, this attitude doesn't change with income. Therefore, if an individual with a low income benefits from a MASP increase in dispersion, he or she will still benefit even with an extremely high income. In addition, the conditions in proposition (3) necessary for nonsearchers to benefit more from a MASP increase in the price dispersion of good 1 can be simplified to $|\varepsilon^h|>s_1$. Again, if this condition holds, it will do so for all income levels.

Finally, one can use the neoclassical utility function and the properties of a(p,q,y) to describe the cost-of-living index version of the formula bias problem in the U.S. consumer price index.[3] If we use base period prices b that differ from q, REK* = E[v(b,1)/v(p,1)]/E[v(b,1)/v(q,1)].

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[3] See Erickson (1995), McClelland (1996) and Reinsdorf (1994b).

Assume that the base prices are unavailable. For example, we may randomly select outlets in the base period without recording prices. We may then wish to impute them using prices q. The bias of this imputation is

$$(10) \qquad \beta = \frac{E\left[v(q,1)/v(p,1)\right]}{E\left[v(q,1)/v(q,1)\right]} - \frac{E\left[v(b,1)/v(p,1)\right]}{E\left[v(b,1)/v(q,1)\right]} = E\left[v(q,1)/v(p,1)\right] - \frac{E\left[v(b,1)/v(p,1)\right]}{E\left[v(b,1)/v(q,1)\right]} .$$

Cross-multiplying and using the definition of covariance, the bias can be written as

$$\beta = \frac{-\operatorname{cov}\left[\frac{v(q,1)}{v(p,1)}, \frac{v(b,1)}{v(q,1)}\right]}{E\left[v(b,1)/v(q,1)\right]} .$$

This is analogous to the example of formula bias given in Erickson (1995). As in that case the bias is likely to be positive because increases in v(q,1) will cause v(q,1)/v(p,1) to rise and v(b,1)/v(q,1) to fall.

VII. Conclusions

Cost-of-living index theory assumes that prices are nonstochastic. Recent work on cost-of-living indexes, however, has developed indexes for the case when prices are randomly distributed. This paper uses consumer surplus theory to show that if utility is set by an indirect utility function in the reference period then two of these indexes are calculated in an identical manner. In this case, prices with stationary distributions can result in these indexes showing an increase in the cost of living. In addition, the index may show an increase in the cost of living even if households are better off than before.

Both of these problems are solved if the index is formed by comparing the level of income with the income necessary to compensate the household for the change in expected utility. This procedure not only resolves these problems, but also creates a context within which researchers may discuss the use of risk aversion in cost-of-living indexes.

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Appendix A

P.1 $a(p,q,y)=e(p,v(q,y))$ is the expenditure function on prices p and utility level u and $v(q,y)$ is the indirect utility function with prices q and income level y. It also implies that for a fixed y and prices $p_{-}= q_{-}$, $\text{sign}(a(p_1,q_1,y)-y)=\text{sign}(p_1-q_1)$, where $p_{-}=[p_2,p_3,...,p_n]$, $q_{-}=[q_2,q_3,...,q_n]$.

P. 2 $a_p(p,q,y) = h(p,v(q,y))$ where $h(p,u)$ is a Hicksian (compensated) demand function. This means that $a_p(p,q,y)$ is increasing in p if demand is positive.

P. 3 $h(p,u)$ is increasing in u for a normal good and $v(q,y)$ is increasing in y.

P. 4 $v(q,y)$ is non-increasing in q and strictly decreasing in q when prices are positive, the individual is unsatiated and demand is positive.

P. 5 $a(q, q, y) = y$.

By P.1 $a(p, q, y) = e(p, v(q, y))$, so that $a(q, q, y) = e(q, v(q, y))$, which is equal to y.

Appendix B

Proposition 1) If the prices for a normal good are random draws from a nondegenerate IID distribution within both periods then EIR($\Omega_{p,q}$,y) > 1.

Proof:

This proof is similar to that of Helms (1984). Without loss of generality let good 1 have random prices in both periods. Let Φ(p) be the nondegenerate cumulative distribution function for the random variables p and q. Then:

$$\frac{E[a(\tilde{q},\tilde{q},y)]}{y} = \frac{\int_B^T\int_B^T a(p,q,y)d\Phi(p)d\Phi(q)}{y}$$

Adding and subtracting y, the numerator becomes:

$$y + \int_B^T\int_B^T [a(p,q,y) - y]d\Phi(p)d\Phi(q)$$

or

$$y + \int_B^T\int_B^{q_1} [a(p,q,y) - y]d\Phi(p)d\Phi(q) + \int_B^T\int_{q_1}^T [a(p,q,y) - y]d\Phi(p)d\Phi(q)\,.$$

By P1., when p < q then y ≡a(q,q,y) < a(p,q,y) and y ≡ a(q,q,y) > a(p,q,y). This implies that the first integral is negative and the second is positive. Whether the numerator is greater or less then y depends upon whether the absolute magnitude of the second integral exceeds the first. That being said, we can cancel out y to get:

$$\int_B^T\int_B^{q_1} a(p,q,y)d\Phi(p)d\Phi(q) + \int_B^T\int_{q_1}^T a(p,q,y)d\Phi(p)d\Phi(q)$$

In this case, the numerator will be greater than y if the second integral exceeds the first. Because p and q are identically distributed we can reverse the order of integration in the second integral.

Doing this and changing the variables of integration, the numerator becomes:

$$\int_B^T \int_B^{q_1} a(p,q,y)d\Phi(p)d\Phi(q) + \int_B^T \int_{p_1}^T a(q,p,y)d\Phi(p)d\Phi(q)$$

Both integrals are being taken over the region p<q, and they are equal at the point p=q, so the relative magnitudes are a function of the slopes of $a(\chi, \xi, y)$ with respect to χ. This depends upon the sign of $a_{\chi\xi}(\chi,\xi,y) = dh(\chi,v(\xi,y))/d\xi$. We can write this out as:

$$dh(\chi,v(\xi,y))/d\xi = h_u(\chi,v(\xi,y))v_\xi(\xi,y).$$

By P3., $h_u(\chi,v(\xi,y))$ is positive for a normal good and by P4. $v_\xi(\xi,y)$ is negative. The slope of $a(\chi,p,y)$ is therefore greater than the slope of $a(\chi,q,y)$ for p<q. This implies that the second integral exceeds the first, so that the numerator exceeds y and REK* > 1.

Proposition 2) Assume the distribution of the price of good 1 is independent of prices $p_2,\ldots,p_m \equiv p\_$, that good one has positive demand and that all prices p_i have positive support $[B_i,T_i]$. *Then a sufficient condition for a MASP increase in the dispersion of p_i to cause $EAK(\Omega_{p,q},y) < 1$ for searchers and non-searchers is* $\rho^{p_1} \geq 0$. *FSS search is itself neither necessary for sufficient). If* $\rho^{p_1} > 0$, *then there exists some distribution function and MASP increase in price dispersion such that $EAK(\Omega_{p,q},y) > 1$ for searchers.*

Proof:

Because $EAK(\Omega_{p,q},y) = \hat{a}(\Omega_{p,q}, y)/y$, $\partial EAK(\Omega_{p,q},y)/\partial z = \hat{a}_z(\Omega_{p,q}, y)$, where z increases the dispersion of the distribution of p, which is equivalent to $\hat{e}_z(\Omega_p,v)$ when the distribution of q is held fixed. Because $\hat{e}(\Omega_p,v)$ varies inversely with the expected indirect utility function, it sufficient to examine the change in expected indirect utility with respect to z. It is also useful to note that, if for all z a function H(p) is positive and strictly decreasing in p over (B,T), then

$$\int_B^T H(p)\Phi_z dp > 0 .$$

If instead H(p) is strictly increasing in p over (B,T) for all z, then:

$$\int_B^T H(p)\Phi_z dp < 0 ,$$

where Φ is the cumulative distribution function.

The expected indirect utility from searching n times is:

$$E(v(\tilde{p},y);z) = \int_{B_1}^{T_1}\ldots\int_{B_n}^{T_n} v(p,y)(1-\Phi)^{n-1} n\phi dp_1 d\phi\_ ,$$

$$= \int_{B_1}^{T_1} E(v(p,y)\big|p_1)(1-\Phi)^{n-1} n\phi dp_1$$

where ϕ_{-} is the pdf of the prices of goods two to m, and n, Φ and ϕ are the numnber of searches, cdf and pdf of good one. Integrating by parts:

$$E(v(p,y);z) = E(v(p,y)|p_1)\Big[-(1-\Phi)^n\Big]\Big|_{B_1}^{T_1} + \int_{B_1}^{T_1} E(v_1|p_1)(1-\Phi)^n\, dp_1$$

where v_1 is the partial derivative of v with respect to p_1. Because $\Phi(T_1)=1$ and $\Phi(B_1)=0$, this may be written as:

B.1) $$E(v(p,y);z) = E(v(p,y)|p_1 = B_1) + \int_{B_1}^{T_1} E(v_1|p_1)(1-\Phi)^n\, dp_1$$

$$= C + \int_{B_1}^{T_1} E(v_1\big|p_1)(1-\Phi)^n\, dp_1$$

Then:

B.2) $$E_z(v(p,y);z) = -\int_{B_1}^{T_1} E(v_1\big|p_1) n(1-\Phi)^{n-1} \Phi_z dp_1 ,$$

where $E_z(v(p,y);z) = \partial E(v(p,y);z)/\partial z$. Because the good has positive demand and all prices are positive, $-E(v_1|p_1)$ is positive. The term $n(1-\Phi)^{n-1}$ is also positive and decreasing. If $\rho^{p_1} \geq 0$ then $-E(v_1|p_1)$ is decreasing in p_1 or is a positive constant. Then $-E(v_1|p_1)n(1-\Phi)^{n-1}$ satisfies the conditions necessary for $E_z(v(p,y);z)$ to be positive so that expected utility increases with dispersion and $\hat{a}(\Omega_p, \Omega_q, y)$ falls. FSS search is clearly not sufficient. Nothing in the above relies on $n_1>1$. FSS search is therefore not necessary. If ρ^{p_1} is negative, then $-E(v_1|p_1)$ is increasing in p_1. If $n(1-\Phi)^{n-1}$ falls more slowly than $-E(v_1|p_1)$ rises then $-E(v_1|p_1)n(1-\Phi)^{n-1}$ rises. Then $E_z(v(p,y);z)$ will be negative and an increase in dispersion raises $\hat{a}(\Omega_{p,q}, y)$.

Proposition 3) Consider the case in proposition 2, and let s_i be the expenditure share for good i. If either $|\varepsilon^h|>s_1(2\eta^y - \rho^y)$ or $2\eta^y - \rho^y > 0$ *then a MASP increase in dispersion benefits nonsearchers more than searchers. If searchers benefit more than nonsearchers, then either* $|\varepsilon^h| \leq s_1(2\eta^y-\rho^y)$ or $2\eta^y-\rho^y\leq 0$ *over some price range.*

Proof:

By the argument in proposition 2), we can consider the expected indirect utility function. By equation (24) of Turnovsky, Shalit and Schmitz (1980), $\mathrm{sgn}(\rho^{p_1})=\mathrm{sgn}(s_1(2\eta^y-\rho^y) - \varepsilon^h)$. Because s_1 is positive and ε^h is negative, if $2\eta^y-\rho^y>0$, then $\rho^{p_1}>0$. We therefore can consider the case where $\rho^{p_1}>0$, which holds if and only if $v_{11}>0$. First, we know that expected utility is higher for searchers than nonsearchers. Using the notation in proposition 1):

B.3) $$E(v|n>1)-E(v|n=1)=\int_B^T E(v|p_1)n[1-\Phi]^{n-1}\phi dp-\int_B^T E(v|p_1)\phi dp>0\,.$$

This holds by differentiating the righthand side of B.1) with respect to n:

$$\frac{d\int_{B_1}^{T_1} E(v_1|p_1)n[1-\Phi]^n dp}{dn}=\int_{B_1}^{T_1} E(v_1|p_1)\ln[1-\Phi]n[1-\Phi]^n dp\,.$$

The terms $E(v_1|p_1)$ and $\ln(1-\Phi)$ are negative and the remaining term is positive, so the right-hand side is positive. Integrating B.3) by parts and collecting terms:

$$E(v|n>1)-E(v|n=1)=\int_B^T E(v_1|p_1)[1-\Phi]\left[(1-\Phi)^{n-1}-1\right]dp$$

Differentiating with respect to z:

B.4) $$\frac{\partial E(v|n>1)-E(v|n=1)}{\partial z}=\int_{B_1}^{T_1} E(v_1|p_1)\left[1-n(1-\Phi)^{n-1}\right]\Phi_z dp$$

The term $[1\text{-}n(1\text{-}\Phi)^{n-1}]$ rises from 1-n to n as p increases from B_1 to T_1. If $v_{11} > 0$, then $E(v_1|p_1)$ rises and so the right-hand side of B.4) is negative. This means that, if a consumer is risk-loving, then increased dispersion brings E(v|n>1) and E(v|n=1) relatively closer together. Because E(v|n>1) is greater than E(v|n=1) and their difference decreases, increasing price dispersion benefits nonsearchers more than searchers. Now suppose that $E(v_1|p_1)$ falls very rapidly over the region over which $[1\text{-}n(1\text{-}\Phi)^{n-1}]$ is negative and equals zero where $[1\text{-}n(1\text{-}\Phi)^{n-1}]$ is non-negative. Then the right-hand side of B.4 will be positive for some increase in dispersion. This shows that it is possible for increases in dispersion to benefit searchers more than nonsearchers. Because it cannot happen if $v_{11} > 0$ over the entire range of prices, it must be the case that $v_{11} \leq 0$ over some range. This only occurs if either $|\varepsilon^h| \leq s_1(2\eta^y\text{-}\rho^y)$ or $2\eta^y\text{-}\rho^y \leq 0$.